Real Yields and the Transmission of Central Bank Balance-Sheet Policies

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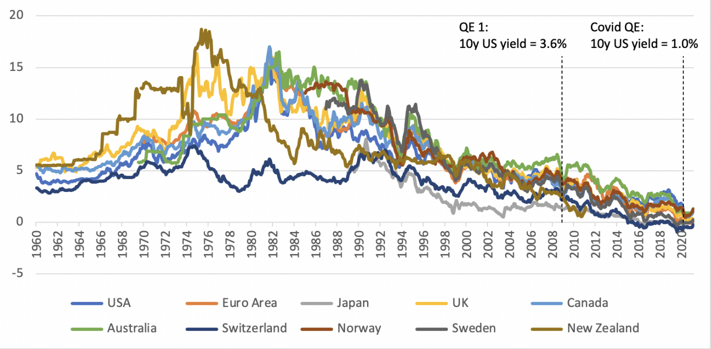
[1] The views expressed here are not official positions of the Chicago Fed or the Federal Reserve System.

Outline

Introduction

Nominal yields have trended downward for 50 years:

Long-term nominal bond yields



Source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis

- Arbitrage implies nominal yields cannot fall below the same ELB that applies to the short rate.
- Does this mean that unconventional monetary policy is doomed?

Introduction

Not necessarily, if QE works through *real* yields.

- Intuitively, real yields should be the operative macroeconomic variables.
  - Gertler & Karadi (2015)
  - Gilcrhist et al. (2015)
- And there is evidence that the real term premium has been the component most affected by QE:
  - Krishnamurthy & Vissing-Jorgensen (2012)
  - Abrahams et al. (2016)
- Real yields are not bounded by an aribitrage argument – can be arbitrarily negative.

This paper

Questions for this paper:

- Can balance-sheet policy still affect real yields—and the macroeconomy—when nominal yields are constrained?
- If so, what type of balance-sheet policy works best?

Framework:

- Macro-finance model of “duration effects” in the yield curve.
- Similar to Greenwood-Vayanos (2014) and Vayanos-Vila (2021).

This paper

I extend the GVV model in several ways:

- Add inflation and real/nominal bond distinction
  - Also see Diez de los Rios (2020)
- Add an ELB on the nominal short rate
  - As in King (2019)
- Let shadow rate follow a Taylor Rule
- Allow for feedback from real yields to inflation and output
  - In the spirit of Ray (2019)

Will allow us to consider the term-structure and macro consequences of various types of balance-sheet policies.

Preview

Results

- *Nominal* QE has no effect once nominal long yields hit the ELB.
- But buying *real* bonds can still be effective.
  - Removes real duration $\rightarrow$ real term premium $\downarrow$
  - Removes an inflation hedge $\rightarrow$ IRP $\uparrow$ $\rightarrow$ real yield $\downarrow$
- Quantitatively, macro effects may be about half the size of past nominal QE operations.

What if there aren't enough real bonds to buy?

- Inflation-indexed term lending program.
- This is equivalent to buying TIPS in the model.

Outline

Outline

No-arbitrage and nominal yields

Why are long-term nominal yields bounded by the ELB?

- Suppose 1-period yield $y^{\$(1)} > 0$ in all periods.
- The 1-period bond price is always $P^{\$(1)} < 1$.
- Would you ever hold a 2-period bond with $P^{\$(2)} > 1$?
- No!
  - Since its price next period < 1 with certainty, you will lose money for sure.
  - No matter what state of the world is realized tomorrow, you will always be better off holding the 1-period bond.
  - Going long 1-period bonds and short 2-period bonds produces risk-free excess returns.
- Same argument extends all the way out the nominal yield curve.
- Note: this is a model-independent result.
  - (See Gagnon and Jeanne, 2020, for a special case.)

No-arbitrage and nominal yields

Formally, if b is a lower bound on $y^{\$(1)}$ and $B \equiv exp(-b)$,

$$\begin{aligned}
P_t^{\$(2)} &= \mathrm{E}_t^{\mathbb{P}}\left[M_{t+1}\frac{P_{t+1}^{\$(1)}}{\Pi_{t+1}}\right] \\
&= \int_S \omega^{\mathbb{P}}(s_{t+1}|s_t)\frac{M(s_t, s_{t+1})P^{\$(1)}(s_{t+1})}{\Pi(s_{t+1})}ds_{t+1} \\
\omega^{\mathbb{Q}}(s_{t+1}|s_t) &\equiv \omega^{\mathbb{P}}(s_{t+1}|s_t)\frac{M(s_t, s_{t+1})}{P^{\$(1)}(s_t)\Pi(s_{t+1})} \\
P_t^{\$(2)} &= P_t^{\$(1)}\int_S \omega^{\mathbb{Q}}(s_{t+1}|s_t)P^{\$(1)}(s_{t+1})ds_{t+1} \\
&\leq P_t^{\$(1)}B \\
y_t^{\$(2)} &\geq (y_t^{\$(1)} + b)/2 \quad \geq \quad b
\end{aligned}$$

No-arbitrage and real yields

Arbitrage argument does not apply to real yields.

- Real short rate is not bounded.
- No matter the value of $P^{(2)}$, there is always a chance $P^{(1)}$ will be higher tomorrow.
- No risk-free arbitrage strategy.

$$y_t^{(2)} \geq \frac{y_t^{(1)} + (b - \mathrm{E}_t^{\mathbb{P}}[\pi_{t+2}])}{2} - \frac{1}{2}\log\left[1 + \frac{\mathrm{cov}_t^{\mathbb{P}}[M_{t+1}, \Pi_{t+2}]}{\mathrm{E}_t^{\mathbb{P}}[\Pi_{t+2}]}\right] + J_t$$

- If marginal utility is high enough in low-r states, $P^{(2)}$ can be arbitrarily big.
- This is how QE will work in the models below.

Outline

Warm-up model

- Nominal short rate i_t is bounded by 0 in all periods.
- Inflation π_{t+1} is known at the beginning of period t.
  - Fisher equation holds for short rates.
- Joint distribution of inflation and short rate next period depends on variance terms σ_i^2, σ_π^2, and $\sigma_{i,\pi}$.
- 2-period nominal and real bonds exist in fixed quantities $x^\$$ and x.
  - Elastic supply of one-period bonds.
- Investors have mean-variance preferences over real return on portfolio, with risk aversion $\frac{a}{2}$.
- Bond prices adjust to clear the market.

A complication...

- Mean-variance investors do not enforce no-arbitrage condition in discrete time.
- **Solution:** Introduce infinitely loss-averse "arbitrageurs."
- Step in only to short $x^{\$}$ (and long 1-period bonds) when ELB would otherwise be violated.
- Arbitrageur demand for nominal 2-period bonds is $z^{\$arb}$.
- They will never hold real bonds.

Bond returns

Excess returns are standard Markowitz portfolio-choice:

$$\begin{aligned} \mathrm{E}[R_1^{\$(2)}] - R_1^{\$(1)} &= a\left((x^{\$} - z^{\$}_{arb})\mathrm{var}[R_1^{\$(2)}] + x\mathrm{cov}[R_1^{\$(2)}, R_1^{(2)}]\right) \\ \mathrm{E}[R_1^{(2)}] - R_1^{(1)} &= a\left((x^{\$} - z^{\$}_{arb})\mathrm{cov}[R_1^{\$(2)}, R_1^{(2)}] + x\mathrm{var}[R_1^{(2)}]\right) \end{aligned}$$

- Bond quantities matter for risk premia depending on the var & covar of real and nominal returns.
- Since today's 2-period bonds are tomorrow's 1-period bonds, the var & covar terms are determined by σ_i^2, σ_π^2, and $\sigma_{i,\pi}$.
- For exposition, assume $x^{\$}$ and x are big enough that arbitrageur demand is zero.

Bond yields

Real and nominal 2-period yields are geometric averages of expected returns:

$$y_0^{\$(2)} \approx \overbrace{\frac{\log[e^{i_0}E[e^{i_1}]]}{2}}^{\text{nom. rate expectation}} + \overbrace{\frac{a}{2}\left[x^{\$}\sigma_i^2 + x(\sigma_i^2 - \sigma_{i,\pi})\right]}^{\text{nom. term premium}}$$

$$y_0^{(2)} \approx \underbrace{\frac{\log[e^{r_0}E[e^{r_1}]]}{2}}_{\text{real rate expectation}} + \underbrace{\frac{a}{2}\left[x^{\$}(\sigma_i^2 - \sigma_{i,\pi}) + x(\sigma_i^2 - 2\sigma_{i,\pi} + \sigma_\pi^2)\right]}_{\text{real term premium}}$$

The multipliers on $x^{\$}$ and x show how nominal and real bond quantities affect nominal and real term premia.

Qualitative results

Result 1 *As long as* $\sigma_{i,\pi}/\sigma_\pi^2 > 1$*, a given amount of nominal QE lowers the long-term real yield by more than the same amount of real QE does.*

$$\begin{aligned}\text{Nom. TP} &= \frac{a}{2}\left[x^\$\sigma_i^2 + x(\sigma_i^2 - \sigma_{i,\pi})\right]\\ \text{Real TP} &= \frac{a}{2}\left[x^\$(\sigma_i^2 - \sigma_{i,\pi}) + x(\sigma_i^2 - 2\sigma_{i,\pi} + \sigma_\pi^2)\right]\end{aligned}$$

Note that this is condition is generally satisfied if the Taylor Principle holds.

Qualitative results

Result 2 *Nominal QE has no effect on either nominal or real yields when the long-term nominal yield is at the lower bound.*

$$\begin{aligned} \text{Nom. TP} &= \frac{a}{2}\left[x^{\$}\sigma_i^2 + x(\sigma_i^2 - \sigma_{i,\pi})\right] \\ \text{Real TP} &= \frac{a}{2}\left[x^{\$}(\sigma_i^2 - \sigma_{i,\pi}) + x(\sigma_i^2 - 2\sigma_{i,\pi} + \sigma_\pi^2)\right] \end{aligned}$$

- σ_i^2 and $\sigma_{i,\pi}$ are zero if we are at the ELB because of expectations.
- (If we are at the ELB because of term premia, nominal QE is completely absorbed by arbitrageurs.)

Qualitative results

Result 3 *The effect of real QE on the long-term real yield is strictly negative, even when nominal yields are at their lower bound.*

$$\begin{aligned}
\text{Nom. TP} &= \frac{a}{2}\left[x^{\$}\sigma_i^2 + x(\sigma_i^2 - \sigma_{i,\pi})\right] \\
\text{Real TP} &= \frac{a}{2}\left[x^{\$}(\sigma_i^2 - \sigma_{i,\pi}) + x(\sigma_i^2 - 2\sigma_{i,\pi} + \sigma_\pi^2)\right]
\end{aligned}$$

- At the ELB, the multiplier on x for real yields is $\dfrac{a\sigma_\pi^2}{2} > 0$.
- Inflation risk premium moves by equal and opposite amount.

Summing up

- Nominal QE is more effective than real QE during "normal" times.
- When nominal yield curve is constrained, nominal QE has no effect.
  - (Note: forward guidance doesn't work here either.)
- But real QE still lowers real yields in this case.
  - Corollary: $\exists$ some $y^{\$(2)}$ below which real QE beats nominal QE.
  - Effectively, Taylor Principle fails near the ELB.
- Intuition:
  - At the ELB, real QE *lowers* real duration risk.
  - At the ELB, real QE *raises* inflation risk.
  - These are equivalent.
- Since real yields matter for the economy, this provides some hope.
- How big might the effects be?

Outline

Quantitative Model

- To quantify this, consider a continuous-time model where
  - investors hold a continuum of real and nominal bonds
  - short rate follows a Taylor Rule w/ELB (shadow rate process)
  - inflation and output depend on long-term real yield
- Core of the model is similar to Vayanos-Vila.
- Nonlinearity means no analytical solution.
- Solve numerically, similarly to King (2019).

State dynamics

Monetary policy:

$$
\begin{aligned}
i_t &= \max[\widehat{i}_t, b] \\
d\widehat{i}_t &= \kappa_i(\mu_t^i - \widehat{i}_t)dt + \sigma_i dZ_t^i \\
\mu_t^i &= r^* + \pi^* + \phi_{i,\pi}(\pi_t - \pi^*) + \phi_{i,g}(g_t - g^*)
\end{aligned}
$$

Output gap:

$$
\begin{aligned}
dg_t &= \kappa_g(\mu_t^g - g_t)dt + \sigma_g dZ_t^g \\
\mu_t^g &= g^* + \phi_{g,\pi}(\pi_t - \pi^*) + \phi_{g,y}(y_t^{(40)} - y^{(40)*})
\end{aligned}
$$

Inflation:

$$
\begin{aligned}
d\pi_t &= \kappa_\pi(\mu_t^\pi - \pi_t)dt + \beta dZ_t^g + \sigma_\pi dZ_t^\pi \\
\mu_t^\pi &= \pi^* + \phi_{\pi,g}(g_t - g^*) + \phi_{\pi,y}(y_t^{(40)} - y^{(40)*})
\end{aligned}
$$

Investor optimization

Real wealth evolves according to

$$dW_t = \int_0^T \left[z_t^{\$}(\tau) \left(\frac{dP_t^{\$(\tau)}}{P_t^{\$(\tau)}} - \pi_t \right) + z_t(\tau) \frac{dP_t^{(\tau)}}{P_t^{(\tau)}} \right] d\tau + \left(W_t - \int_0^T \left[z_t^{\$}(\tau) + z_t(\tau) \right] d\tau \right) r_t dt \tag{1}$$

Taking W_t as given, investors choose $z_t^{\$}(\tau)$ and $z_t(\tau)$ to solve

$$\max_{\{z_t^{\$}(\tau), z_t(\tau)\} \forall \tau} \mathsf{E}_t [dW_t] - \frac{a}{2} \mathsf{var}_t [dW_t]$$

subject to (1).

Market clearing:

$$z_t^{\$}(\tau) = x_t^{\$}(\tau)$$

$$z_t(\tau) = x_t(\tau)$$

- A solution is a set of state-contingent bond prices that clears the market at each t.
- Assume $x_t^{\$}(\tau) = x^{\$}$ and $x_t(\tau) = x$ for all t and τ.
- (This probably doesn't matter much.)

Parameter values

Description	Parameter	Value	Calibrated to…
Inflation target	π^*	2.4%	Average long-run BC CPI forecast
Inflation inertia	$\exp(-\kappa_\pi)$	0.51	Estimated state-space model*
Inflation response to lag 10Y real yield ι	$\phi_{\pi,y}$	**-0.086**	Estimated state-space model*
Inflation response to lag GDP gap	$\phi_{\pi,g}$	0.018	Estimated state-space model*
Inflation innovation std. dev.	σ_π	0.37%	Estimated state-space model*
Effective lower bound	b	0%	Assumed zero
Shadow rate inertia	$\exp(-\kappa_i)$	0.76	Carlstrom & Fuerst (2008)
Shadow rate target response to inflation	$\phi_{i,\pi}$	1.5	Taylor (1993)
Shadow rate target response to GDP gap	$\phi_{i,g}$	0.5	Taylor (1993)
Shadow-rate innovation std. dev.	σ_i	0.30%	Estimated state-space model*
Output gap inertia	$\exp(-\kappa_g)$	0.87	Estimated state-space model*
Output gap response to lag 10Y real yield	$\phi_{g,y}$	**-0.08**	Estimated state-space model*
Output gap response to lag inflation	$\phi_{g,\pi}$	0.17	Estimated state-space model*
Output gap innovation std. dev.	σ_g	0.56%	Estimated state-space model*
Inflation response to output gap innovation	β	0.16	Estimated state-space model*
Risk aversion	a	1	Normalization

- Taylor Rule parameters take standard values.
- Other dynamic parameters are based on estimated model over 1999 - 2020.
- Remaining parameters are calibrated to specific interest-rate scenarios.

Experiment

Check how a given quantity of QE affects yields, inflation, and output

- when directed to real vs. nominal bonds
- in different initial interest-rate environments

Specifically, the baseline “shock” is a change in $x^{\$}$ that lowers $y^{\$(40)}$ by 100bp in an environment similar to 2008-9.

- What does this same shock do if it hits x instead?
- How do the effects differ in a 2020-like environment?
- How do the effects differ in an even-*lower* r* environment?

Specification of three scenarios

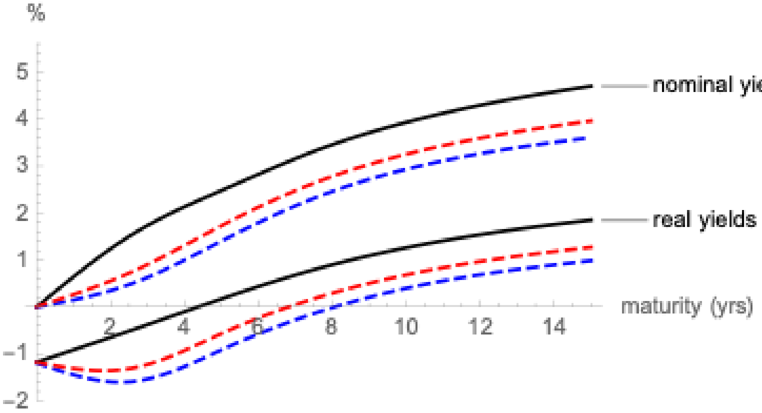
	High rate (Similar to 2008-9)	Moderate rate (Similar to 2020)	Low rate (Hypotehtical)
Inflation (π_0)	1%	1%	1%
Output gap (g_0)	-4%	-4%	-4%
Shadow rate ($\hat{\imath}_0$)	0%	0%	0%
Eq. real short rate (r^*)	1.7%	0%	-1.7%
Nominal bond parameter ($x^\$$)	0.47	-0.03	-0.03
Real bond parameter (x)	-0.69	0.12	0.12
10y nominal yield ($y_0^{\$(40)}$)	4.3%	1.4%	0.0%
10y real yield ($y_0^{(40)}$)	1.6%	-0.6%	-1.0%

A complication...

- Since economy improves with QE, policy rate rises endogenously over time.
- Expectations for $\{r_{t+s}\}$ jump at time-t.
- Increase in expectations component of real yield offsets some of the decrease in term premium.
- **Solution:** "Neutralize" this feedback with a shock to $\widehat{i}_t$ such that the expectations component of $y^{\$(40)}$ remains unchanged.
- (Also consider case where short-rate feedback is allowed.)

Yield curve responses - baseline

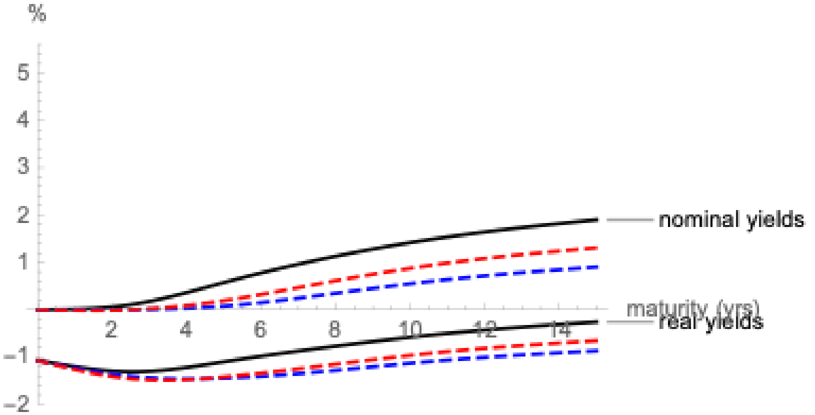
High-rate scenario



Black - initial level Blue - nominal QE Red - real QE

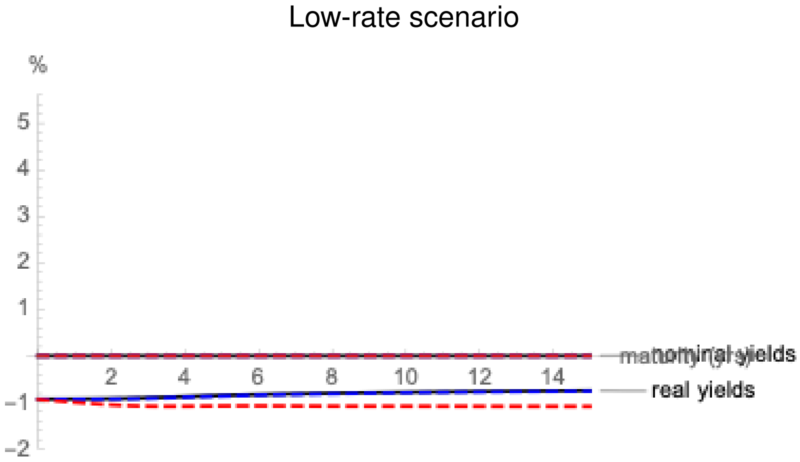
Yield curve responses - baseline

Moderate-rate scenario



Black - initial level Blue - nominal QE Red - real QE

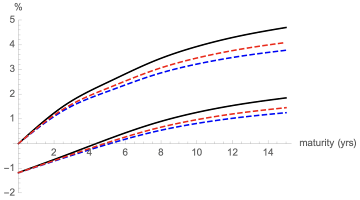
Yield curve responses - baseline



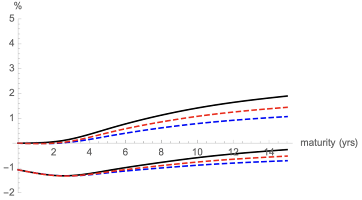
Black - initial level Blue - nominal QE Red - real QE

Yield curve responses - allowing feedback

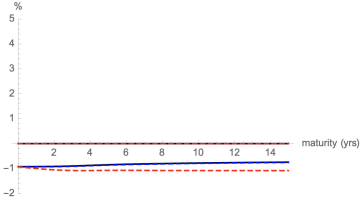
high rate



moderate rate

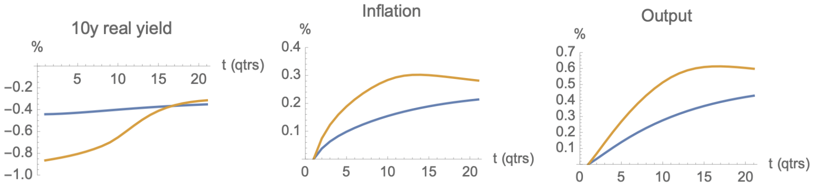


low rate

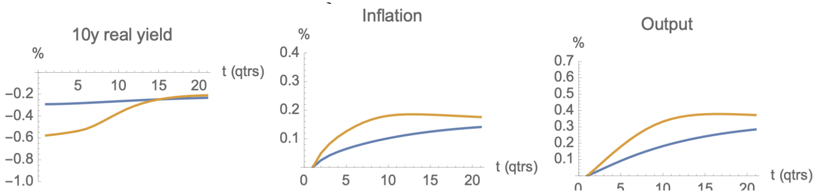


Impulse-response functions – “high rate”

Nominal QE



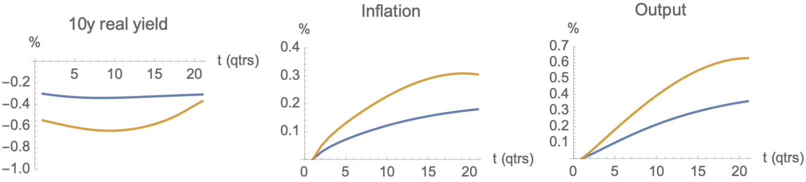
Real QE



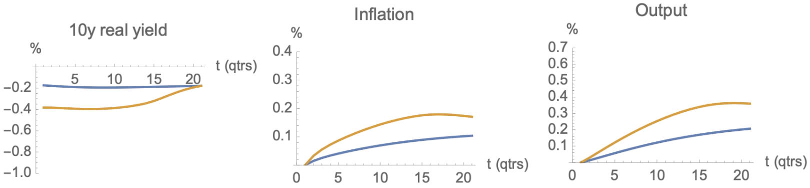
Gold - baseline Blue - feedback allowed

Impulse-response functions – “moderate rate”

Nominal QE



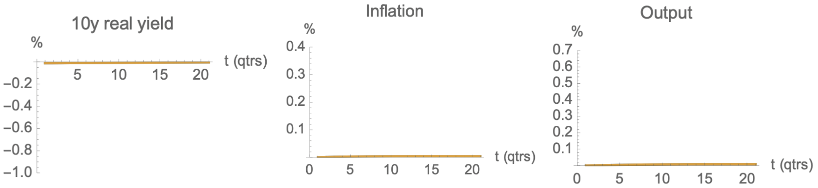
Real QE



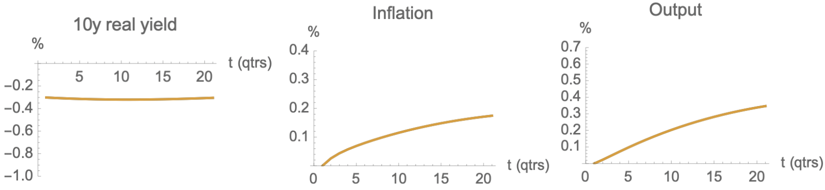
Gold - baseline Blue - feedback allowed

Impulse-response functions – “low rate”

Nominal QE



Real QE



Gold - baseline Blue - feedback allowed

Summary of scenario analysis

	High rate (Similar to 2008-9)		Moderate rate (Similar to 2020)		Low rate (Hypotehtical)	
	Nom. QE shock	Real QE shock	Nom. QE shock	Real QE shock	Nom. QE shock	Real QE shock
Initial effect on yields (bp)						
$y^{\$(40)}$	-100	-68	-87	-54	0	0
$y^{(40)}$	-86	-58	-55	-38	0	-30
10y infl. comp.	-14	-10	-32	-16	0	+30
Dynamic effect on macro variables (bp)						
π_{10}	+29	+18	+24	+15	0	+12
π_{20}	+28	+18	+30	+17	0	+17
g_{10}	+54	+35	+30	+27	0	+22
g_{20}	+60	+37	+63	+36	0	+35

- Similar for “feedback allowed.”
- Also similar when we include signaling channel of QE.

Outline

Inflation-indexed term lending

Economically equivalent to real QE:

- Inflation exposure transferred from gov't to private sector
- Size and real duration of CB balance sheet $\uparrow$
- Reserves $\uparrow$ by same amount

From a bank's perspective:

- Nominal interest expense rises with inflation, but interest income does not.
- Would require lower expected real rate on loan to accept this risk.
- Incentives to pass through to real sector through
  - Inflation-indexed loans
  - Derivatives
  - Purchasing inflation-hedging assets

Inflation-indexed term lending

Advantages over negative nominal short rates:

- Can't arbitrage by hoarding cash
- No adverse effects on short-term investors (MMMFs)
- No "reversal rate" problem

In practice:

- This has to be *term* lending to be effective.
- Fed would require 13(3) authority.
- But other central banks have done nominal lending at term recently.

Outline

Conclusion

- Theory and evidence suggest *real* yields matter for the economy.
- If nominal yields remain low, forward guidance and nominal QE may be impotent in future recessions.
- But *real* QE—or equivalent operations—can still work.
- Macro effects of such programs might be about half as big as those of past nominal QE programs.
- Academic contributions - extend no-arb model of bond supply in term structure to incorporate:
  - Inflation
  - Real/nominal bond distinction
  - ELB on nominal rates
  - Real activity
  - Monetary policy rule
  - Feedback from yields to economy
  - Realistic parameter values

Thanks!