Duration Effects in Macro-Finance Models of the Term Structure

Thomas B. King
Federal Reserve Bank of Chicago[1]

June 16, 2019

[1] The views here do not represent those of the Chicago Fed or the Federal Reserve System.

Introduction

Evidence suggests that debt quantities affect the yield curve, asset prices, and perhaps the macroeconomy.

- Monetary policy has tried to exploit this link through QE programs.
- These "duration effects" are not well understood, and are absent from most models.

Coherent models of duration effects exist, but they are far removed from the rest of macro-finance.

- Many open issues remain.
    - Interactions with macro variables.
    - Effects of nonlinearities.
    - Quantifying effects.

Introduction

I try to make some progress on these issues.

- General framework for incorporating duration effects in macro-finance models.
  - Key feature: SDF depends on return on wealth.
- Method for solving such models

Outline of talk:

- Literature review
- Conceptual framework
- Solution algorithm
- Illustrations in one-factor model
- Estimated four-factor model

First-generation portfolio balance

Frankel (1985):

$$\max_{\mathbf{w}} \mathrm{E}_t[R_{t+1}] + \frac{a}{2}\mathrm{var}_t[R_{t+1}]$$

FOC implies implicit demand condition:

$$\mathrm{E}_t[\boldsymbol{\rho}_{t+1}] = i_t + a\Sigma\mathbf{w}_t$$

Asset supply $\mathbf{w}_t^s$. In equilibrium:

$$\mathbf{w}_t = \mathbf{w}_t^s$$

By changing $\mathbf{w}_t^s$, a policymaker can change returns.

Versions of this model still appear occasionally. (Bernanke et al., 2004; Joyce et al., 2011; Neely, 2012)

Problems:
- Σ exogenous
- Not forward-looking
- No solution for bond prices / yields

Portfolio balance in the term structure

Vayanos and Vila (2009) and Greenwood and Vayanos (2014) solve a version of this problem for a portfolio of bonds:

$$\max_{\mathbf{w}} \mathrm{E}_t[W_{t+1}] + \frac{a}{2}\mathrm{var}_t[W_{t+1}]$$

FOC implies implicit demand condition:

$$\mathrm{E}_t[\boldsymbol{\rho}_{t+1}] = i_t + a\Sigma\tilde{\mathbf{w}}_t$$

where $\tilde{\mathbf{w}}_t = W_t\mathbf{w}_t$.

If $\tilde{\mathbf{w}}_t^s$ and i_t follow affine processes, a solution exists under some conditions.
- Under stronger conditions, prices can be found analytically.

E.g., if $\tilde{\mathbf{w}}_t^s$ is constant,

$$y_t^{(N)} = \frac{1}{N}\sum_{i=1}^{N}\mathrm{E}_t[i_{t+i}] + a\left(\mathbf{1}'\Sigma\tilde{\mathbf{w}}^{\mathbf{s}}\right)$$

More on GVV

This framework seems to capture the "duration channel."

- Smaller or shorter-duration $\tilde{\mathbf{w}}_t^s$ lowers yields.
- Arbitrage-free. Nothing "special" about bonds.
- Pointed to in many empirical papers on QE.
- Expanded by e.g., Hamilton and Wu (2012), Altavilla et al. (2015), Greenwood et al. (2015), Kaminska and Zinna (2015), Haddad and Sraer (2015), Malkhozov et al. (2016), and King (forthcoming).

But it is hard to find these effects in standard models.

Consumption-based models do not have a role for debt structure:
Utility:

$$U_t = \sum_{i=1}^{\infty} \beta^i u(c_{t+i})$$

SDF:

$$M_{t,t+n} = \beta^n \frac{u(c_{t+n})}{u(c_t)}$$

Since debt does not appear here, it has no affect on asset prices.

Eggertsson and Woodford (2003) - It has no affect on anything else either.

Reconciling these views

The key aspect of GVV is that the pricing kernel depends on the return on wealth:

$$M_{t,t+1} = \exp\left[-i_t + a(R_{t+1} - \mathrm{E}_t[R_{t+1}])W_t\right]$$

Eggertsson-Woodford assume this away. But it is not hard to find models where it occurs:

- Epstein-Zin
- Kurz (1968); Michaillat and Saez (2016)
- Intermediary asset pricing

The goal of the paper is to explore the potential for duration effects in such circumstances.

Difficulties

These models are almost always nonlinear.

GVV bends over backwards to get linearity:

- Investors have *absolute* risk aversion.
- *Market* values of debt are exogenous.
- No ELB.
- No interactions with macro factors.

To make progress, we need to move beyond models with analytical solutions.

Setup

Let

$$\mathbf{p}_t = \begin{pmatrix} p_t^{(1)} & \ldots & p_t^{(N)} \end{pmatrix}$$

$$\mathbf{q}_t = \begin{pmatrix} 1 & p_t^{(1)} & \ldots & p_t^{(N-1)} \end{pmatrix}$$

$$R_{t+1} \equiv \frac{\mathbf{X}_t'\mathbf{q}_{t+1}}{\mathbf{X}_t'\mathbf{p}_t}$$

where the $X_t^{(n)}$ is net claims/exposures at maturities $n = 1, \ldots, N$.
- Note: only shares, not absolute quantities, matter.

Let the 1-period SDF be

$$M_{t,t+1} = M(\mathbf{s}_{t+1}, \mathbf{s}_t, R_{t+1})$$

Bond prices are:

$$p_t^{(n)} = \mathrm{E}_t \left[M_{t,t+n} \right]$$

This sets up a nonlinear recursion.

Solution method

Assume the transition density $\tau\left(\mathbf{s}_t + 1|\mathbf{s}_t\right)$ and the real SDF $M(.)$ are known.
- Nominal SDF $M^{\$}_{t+1} = \Pi_{t+1} M_{t+1}$.

Initialize:

- Discretize the state space.
- Guess bond prices at each node.

Iterate on the following:

- Calculate R_{t+1} between each pair of nodes.
- Calculate $M_{t,t+1}$ and $M^{\$}_{t,t+1}$ between each pair of nodes.
- Calculate bond prices at each node.

Warm-up model

Consider the one-factor model in which investors have myopic preferences over wealth:

$$\max_{\mathbf{X_t}} \mathrm{E}_t \left[\delta_t W_{t+1}^{\lambda+1} \right]$$

Let:

$$i_t = \phi_0 + \phi_1 i_{t-1} + e_t$$

$$X_t^{(n)} \propto \mathrm{N}\left[z, 1 \right]$$

SDF:

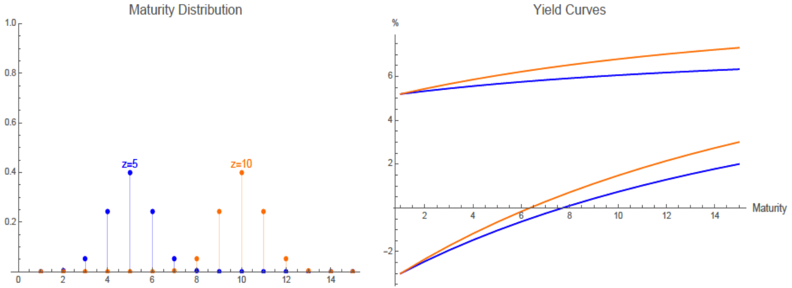
$$M_{t+1} = \delta_t R_{t+1}^{\lambda}$$

where

$$\delta_t = \frac{\exp\left[-i_t \right]}{\mathrm{E}_t \left[R_{t+1}^{\lambda} \right]}$$

Parameters calibrated to post-1971 yields.

Yield curves for different values of z and i:



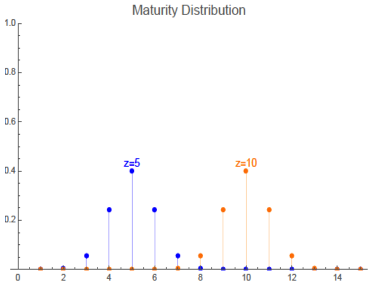
Effect of the ELB

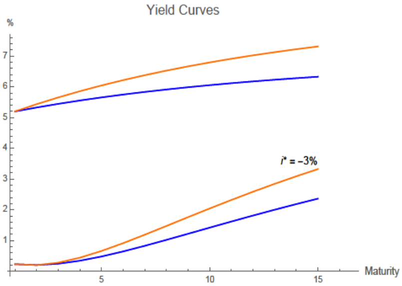
Now instead suppose

$$i_t^* = \phi_0 + \phi_1 i_{t-1}^* + e_t$$

$$i_t = \max[i_t^*, b]$$

Yield curves for different values of ζ and i^*:



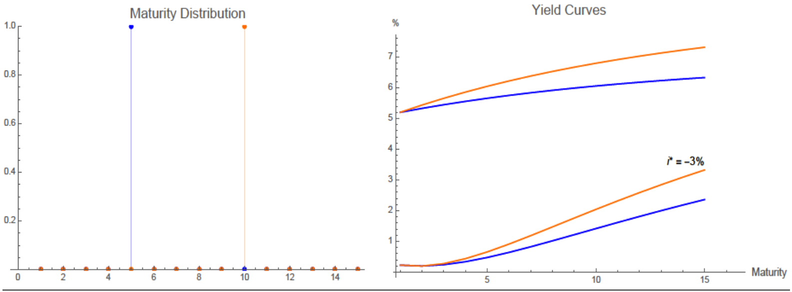


Effect of the asset distribution

Now instead suppose

$$X_t^{(n)} =\propto \mathrm{N}[z, 0.1]$$

Yield curves for different values of ζ and i^*:



A more-serious model

SDF:

$$M^{\$}_{t,t+1} = \delta_t \Pi_{t+1} G^{\lambda^G}_{t+1} R^{\lambda^R}_{t+1}$$

Asset distribution:

$$X^{(n)}_t \propto \mathrm{N}\left[z_t, 1\right]$$

State dynamics:

$$\pi_t = \phi^{\pi}_0 + \phi^{\pi}_1 \tilde{\mathbf{s}}_{t-1} + e^{\pi}_t$$

$$g_t = \phi^{g}_0 + \phi^{g}_1 \tilde{\mathbf{s}}_{t-1} + e^{g}_t$$

$$i^*_t = \phi^{i^*}_0 + \phi^{i^*}_1 \tilde{\mathbf{s}}_{t-1} + \phi^{i^*}_2 i^*_{t-1} + e^{i^*}_t$$

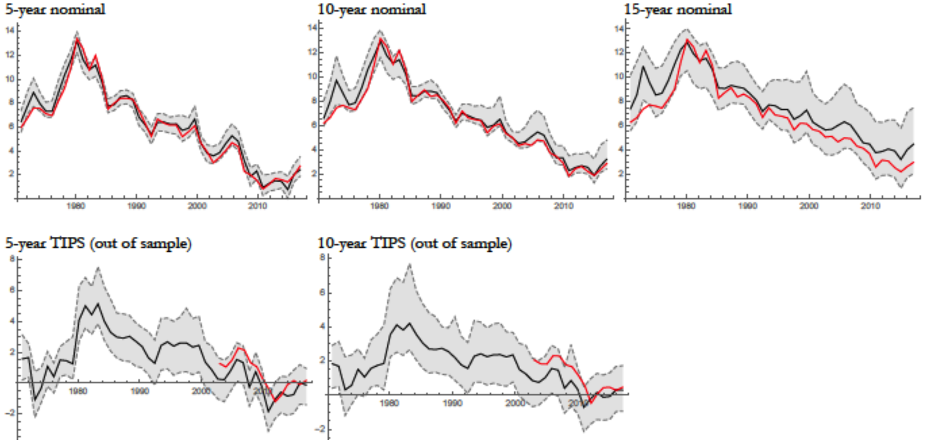
$$z_t = \phi^{z}_0 + \phi^{z}_1 \tilde{\mathbf{s}}_{t-1} + \phi^{z}_2 z^*_{t-1} + e^{z}_t$$

where $\tilde{\mathbf{s}}_t = \left(\pi_t, g_t, i_t, y^{\$(10)}_t\right)$.

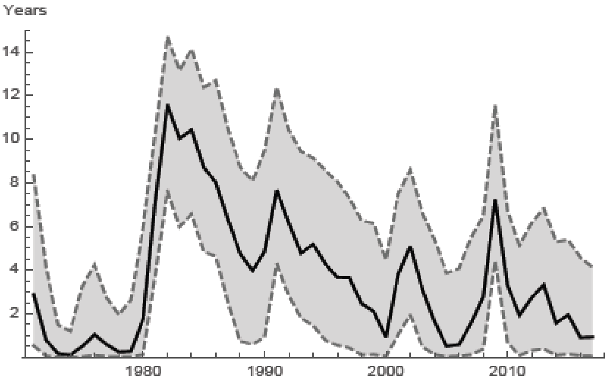
Estimation

- Data on inflation, consumption, and GSW nominal yields since 1971.
- *Annual* data captures lower frequencies and helps computationally.
- z_t and i_t^* are treated as a latents factor and estimated with a particle filter.
- Fixed parameters are estimated by Gibbs.

Fit

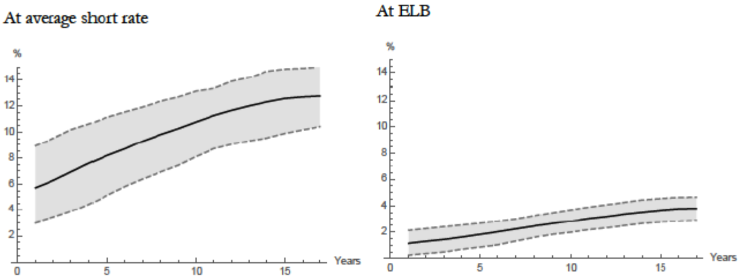

5-year nominal
10-year nominal
15-year nominal
5-year TIPS (out of sample)
10-year TIPS (out of sample)
1980
1990
2000
2010

Estimated duration factor

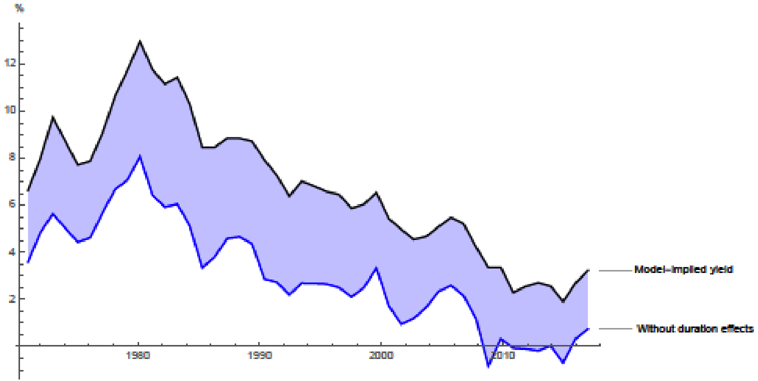


Regression of this series on Treasury WAM has coefficient of 1.36***.

10-year yield as function of duration



Contribution of duration to 10-year yield



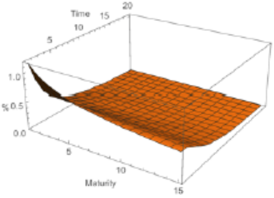
Impulse-response functions

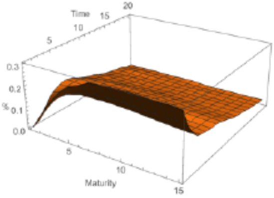
- Interpolate quarterly values of z_t, and re-run model to obtain higher frequency error covariances.
- Impose short-run ordering, with monetary-policy shocks second to last and duration shocks last.

Impulse-response functions: yield curves

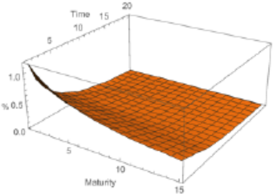
Monetary-policy shock *Duration shock*

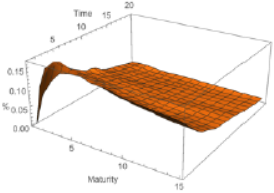
<u>Nominal yields</u>



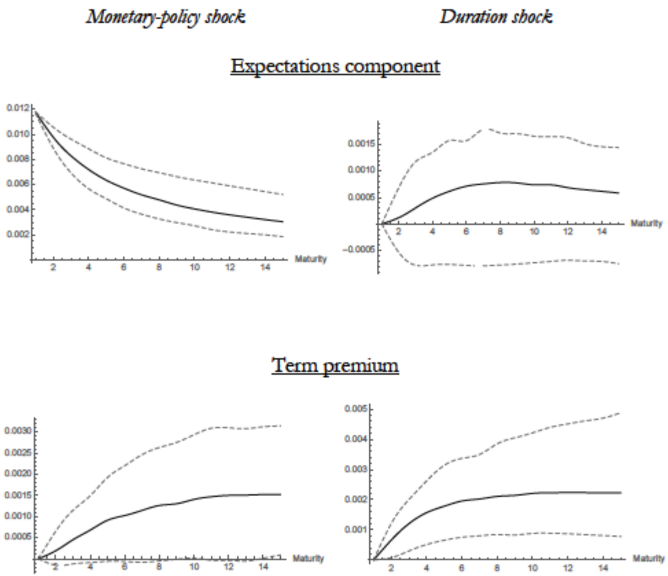


<u>Real yields</u>

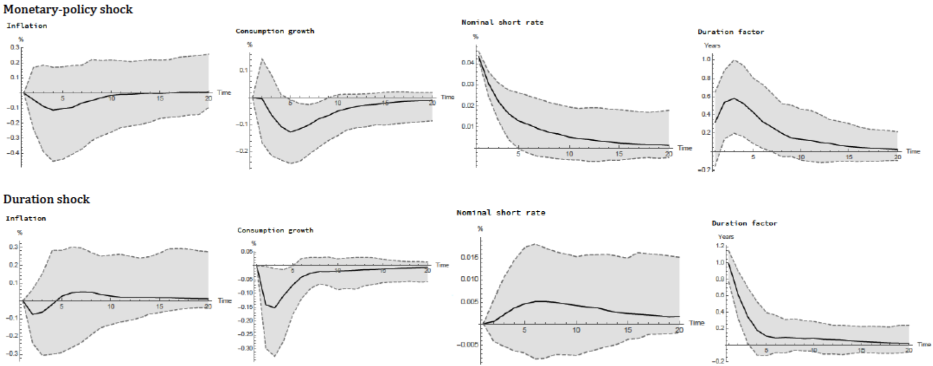




Impulse-response functions: decomposition



Impulse-response functions: macro variables



Conclusion

Duration effects have been important, but models of these effects are absent from standard macro-finance models.

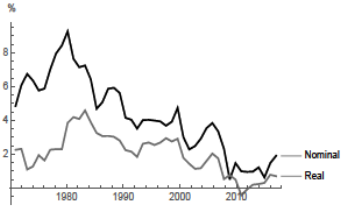
- This paper takes a first step in reconciling these literatures.
- Results suggest that effects of duration shocks are are small, but duration exposures could nonetheless be important for explaining term premia.

There is much more to be done.

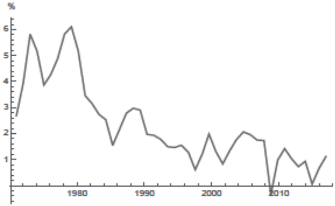
- Further elaboration of pricing kernels.
- Incorporating such effects into fully specified macro models (DSGE).

Appendix: Decomposition of 10-year yield

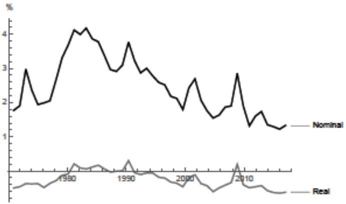
Expected short rates



Expected inflation



Term premia



Inflation-risk premium

Appendix: Fit

Table 3. Fit Statistics

	Baseline	Baseline parameters, but $\lambda^R = 0$	Re-estimated w/o R_{t+1} in SDF
Inflation t+1	1.37%	1.37%	1.38%
Consumption t+1	1.20%	1.20%	1.20%
1y yield t+1	1.42%	1.42%	1.44%
5y nom. yield t	0.30%	3.51%	0.60%
10y nom. yield t	0.25%	2.98%	0.49%
15y nom. yield t	0.27%	3.28%	0.58%

Appendix: Convergence in 1-factor model