Monetary-Policy Risk and Equilibrium Asset Prices

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June 1, 2026

Abstract

Nominal yields, real yields, stock returns, and implied volatilities all vary more on days with FOMC communication than on FOMC "blackout" days and all respond significantly to changes in interest-rate uncertainty around FOMC announcements. These facts imply that news about the first and second moments of monetary policy are important, and priced, sources of risk. A New Keynesian model with stochastic variation in the policy rule quantitatively matches this evidence. The model shows that monetary-policy risk contributes significantly to excess returns on equities and nominal bonds and more than accounts for the risk premium on inflation-indexed bonds. Transitory increases in monetary-policy risk lower stock and bond prices, reduce inflation-risk premia, and generate macroeconomic weakness similar to that produced by aggregate-demand shocks. The effective lower bound on interest rates reduces the size of monetary-policy risk but magnifies its asset-pricing consequences, consistent with the data. A shock to monetary-policy risk of the size that occurred in 2022 - 2023 raises 10-year real yields by 75 basis points and reduces equity prices by 12%.

*Corresponding author, thomas.king@chi.frb.gov. For helpful comments and discussions, we thank Gadi Barlevy, Brent Bundick, Stefania D'Amico, Anthony Diercks, Francois Gourio, Carolin Pflueger, and seminar participants at the Federal Reserve Bank of Chicago. Albert Chen provided excellent research assistance. The views expressed here do not reflect official positions of the Federal Reserve.

1 Introduction

When central banks send signals about the course of policy that differ from what investors expect, financial markets react strongly. Indeed, a large empirical literature shows that surprise innovations in monetary policy and its expected path have large effects on both stock and bond prices (Rigobon and Sack (2004); Bernanke and Kuttner (2005); Bauer and Swanson (2023)). Given that policy shocks are associated with large movements in asset returns, one should expect investor uncertainty about how the central bank may depart from its rule to contribute significantly to risk premia. In this paper, we study the implications of uncertainty about policy behavior—what we call "monetary-policy risk"—for asset prices.

To take a concrete example, monetary-policy risk (MPR) appears to have risen dramatically during the post-COVID tightening of policy in the United States. Panel A of Figure 1 plots the standard deviation of the one-year-ahead federal funds rate calculated from subjective probability distributions reported by large dealer banks in each December.[1] Between year-end 2021 and year-end 2023, this measure of uncertainty about the path of monetary policy more than tripled. Meanwhile, Panels B and C show that uncertainty about short- and medium-term inflation was essentially unchanged between year-end 2021 and 2023, while uncertainty about one-year-ahead GDP declined. Thus, the increase in uncertainty about interest rates was not a mechanical passthrough of uncertainty about the macroeconomy via the monetary-policy reaction function; rather, it reflected an increase in uncertainty about the course of monetary policy *conditional* on the state of the economy.[2]

[1] Federal Reserve Bank of New York, Survey of Primary Dealers. Data available at https://www.newyorkfed.org/markets. Dealers report probabilities of outcomes within bins, and we calculated the standard deviation using the midpoint of each bin, averaging the responses across dealers. Collection of these data has been inconsistent over the years, resulting in the missing observations of the fed funds rate uncertainty in 2012 and 2013 and shorter series lengths for inflation and GDP.

[2] Contemporary commentary was consistent with this interpretation. See, for example, Greg Ip, "Fed's Rate Increases Defy all the Rules," *The Wall Street Journal*, Oct. 5, 2022. In principle, a

We show empirically and theoretically how shocks to MPR like this one can be important drivers of financial returns. Heuristically, there are at least three channels through which we should expect MPR shocks to pass through to asset prices. First, because they increase volatility and downside risk, such shocks lead to higher risk premia across assets. Second, because they induce macroeconomic weakness through precautionary behavior (and perhaps through other channels), such shocks lower equity prices by reducing expected cash flows. Third, if monetary policy itself responds to the macroeconomic deterioration, positive MPR shocks may be associated with a lower expected path of risk-free rates, and the resulting discounting effects may offset part of the deterioration in both stock and bond prices. The jump in MPR in 2022 indeed coincided with increases in estimates of real and nominal term premia and a significant decline in the stock market.[3] Although the FOMC was raising the federal funds rate during this period to fight inflation, it likely did so by less than it otherwise would have because the increase in MPR was itself exerting some restraint on the economy.

Over longer periods, we argue that MPR contributes importantly to the general levels of risk premia on stocks and bonds, shedding light on long-standing questions about why the excess returns on those assets are so large (Mehra and Prescott (1985), Backus, Gregory, and Zin (1989)). The intuition is straightforward: shocks to monetary policy move the payoffs on both stocks and bonds in the same direction as real activity. More uncertainty about such shocks—i.e., a higher level of MPR—therefore increases the likelihood that these assets will experience bad returns in bad states of the world, and risk-averse investors require greater compensation if that risk is high. Of particular note, MPR can also explain the positive average premium, not just on nominal bonds,

perceived increase in the magnitude of reaction-function coefficients, such as that estimated by Bauer, Pflueger, and Sunderam (2024), could also raise interest-rate uncertainty without an increase in macro uncertainty. However, we show in Section 6 that this effect cannot account for the magnitudes shown in the figure.

[3]See Cieslak, McMahon, and Pang (2025) for a discussion of the nominal term premium during this period.

but on *inflation-indexed* bonds, a feature of the data that consumption-based asset-pricing models have struggled to match (Hsu, Li, and Palomino (2021)).

We begin by quantifying the size and behavior of MPR in the data since 1994. Our identification relies on two separate analyses. First, we gauge the size of MPR by comparing asset-price volatility on days with and without FOMC communication. Specifically, during FOMC "blackout periods," Fed officials and staff refrain from speaking to the public about matters having to do with monetary policy. Thus, effectively, there is no monetary-policy risk on these days. We show that asset prices are uniformly more volatile on non-blackout days, implying that MPR adds nontrivially to risk across markets. For example, based on the differences in volatility between blackout and non-blackout days, we estimate that MPR contributes about 3 percentage points (out of a total of 8.5%) to the quarterly standard deviation of stock prices and 17 basis points (out of 41) to the quarterly standard deviation of the 10-year nominal yield. Second, we expand the event-study evidence of Bauer, Lakdawala, and Mueller (2022), who computed the responses of various financial variables to changes in short-term interest-rate uncertainty on the days of FOMC announcements. This exercise identifies the passthrough of changes in MPR to stock prices, real and nominal bond yields, and the implied volatilities of equities and interest rates. We confirm the Bauer et al. result that these multipliers are economically large and statistically significant.

We make quantitative sense of these results through the lens of a nonlinear New Keynesian (NK) asset-pricing model. We allow for MPR in the model by adding persistent shocks with stochastic volatility to the monetary-policy reaction function. The shock volatility captures uncertainty about random deviations from the policy rule (i.e., uncertainty about standard NK "monetary-policy shocks") and also approximates to a first order uncertainty about transitory changes in the parameters of the rule itself. We calibrate the size and persistence of the shocks, and the size and persistence of

their volatility, to match the event-study and blackout-period evidence just discussed. Notably, the model can quantitatively account for the empirical responses of asset prices to MPR shocks as well as to standard monetary-policy shocks.

With parameters that thus match the data, the level and variation in MPR in the model explains 60 basis points of the average nominal ten-year term premium and contributes about 170 basis points to the average equity premium. It implies an average real ten-year term premium of 95 basis points—more than explaining the average value typically estimated in the data.[4] Near the steady-state, a transitory increase in MPR generates the following outcomes: (1) implied and realized volatilties rise across asset classes; (2) output and inflation fall due to precautionary behavior; (3) the equity premium rises, which, together with the lower expected nominal output, causes stock prices to fall; (4) real and nominal term premia increase, but these effects are smaller than the decline in expected short-term interest rates in response to the deteriorating economy; thus, (5) the nominal yield curve steepens, and long-term nominal bonds realize negative excess returns; (6) on the other hand, the real yield curve moves higher; (7) the inflation risk premium falls.

When we allow for a zero lower bound in the model, the impact of MPR on interest-rate volatility is damped, so shocks to MPR generally have smaller effects on risk premia. However, the effects on realized stock and real-bond returns are larger at the ELB because the inability of nominal interest rates to respond amplifies the drag on output and inflation. To test these predictions in the data, we return to the event-study, dividing the sample into observations when policy was constrained by the ELB and observations when it was not. We show that, as the model predicts, the relationships between short-rate uncertainty and asset prices are larger and more statistically signif-

[4]For example, over the period covered by our data, the model of D'Amico, Kim, and Wei (2018) estimates an average 10-year real term premium of 63 bp. This observed value is likely lower than the value we attribute to MPR shocks because of other kinds of risk. In particular, risk to the demand side of the economy generally has a negative effect on real term premia.

icant at the ELB. Indeed, our model generally matches the quantitative responses of most asset prices to MPR shocks in both non-ELB and ELB sub-samples.

As a case study, we use the model to examine the increase in MPR that occurred around the rapid policy tightening of 2022. Introducing shocks to replicate the increase in interest-rate uncertainty shown in Figure 1, and accounting for the effects of liftoff from the ELB in 2022, we estimate that shocks to MPR added about 75 basis points to the 10-year real term premium, pushed up the VIX by about 8 points, and caused a cumulative decline in equity prices of about 12 percent. The model implies that the 10-year inflation-risk premium on nominal bonds fell about 30 basis points in response to this shock.

Our paper connects to three broad strands of literature. First, recent work by Husted, Rogers, and Sun (2020), Bauer et al. (2022), and Bundick, Herriford, and Smith (2022) has shown that uncertainty about interest rates and monetary policy varies over the business cycle and even over the FOMC cycle. These empirical measures of uncertainty capture the total variance of policy rates rather than isolating the exogenous component of policy that we are interested in; however, it is likely that some portion of the fluctuations documented in these studies is due to changes in the public's uncertainty about whether monetary policymakers might at some point deviate from their rule or change the rule itself. Meanwhile, Cieslak and Pang (2021) and Kroencke, Schmeling, and Schrimpf (2021) provide evidence of changes in risk premia around FOMC announcements that are consistent with important changes in monetary-policy risk. Our modeling helps to make sense of these results and to connect them to broader asset-pricing theory.

Second, our results are related to an expansive empirical and theoretical literature that attempts to characterize the effects of uncertainty on business cycles, including Bloom (2009), Gilchrist, Sim, and Zakrajsek (2014), Jurado, Ludvigson, and Ng (2015),

Basu and Bundick (2017), Bloom et al. (2018), and many others. Some of these papers consider macroeconomic or financial uncertainty in reduced form, rather than homing in on specific structural sources; those that are specific tend to focus on fiscal or supply-side sources, rather than on monetary policy. One exception is Mumtaz and Zanetti (2013), who embed stochastic volatility in the policy reaction function in a structural model in a way that is similar to ours. However, like most of this literature, they do not examine asset-pricing implications. While uncertainty shocks of different structural types tend to have similar macroeconomic effects in theoretical models, they can have much different consequences for asset prices, as we discuss below.

The third set of papers we build on are those that seek to understand asset prices in the context of general-equilibrium models, and in sticky-price frameworks in particular. Papers such as Hordahl, Tristani, and Vestin (2008), Bekaert, Cho, and Moreno (2010), Li and Palomino (2014), Rudebusch and Swanson (2012), Binsbergen et al. (2012), Dew-Becker (2014), Kung (2015), and Swanson (2021) have closed much of the gap between asset pricing and modern macroeconomic theory. Yet, in all of these models, monetary-policy shocks are treated as homoskedastic or are absent altogether. Within this literature, the papers that are perhaps methodologically closest to ours are Gourio and Ngo (2020) and Pflueger (2025). Gourio and Ngo show how the proximity of the ELB in a New Keynesian model endogenously lowers term premia and changes the correlation of stock returns and inflation from negative to positive in the presence of productivity shocks. Pflueger identifies changes in the monetary policy rule, in conjunction with a declining role for technology shocks, as the reason for the decline in the stock-bond correlation over time. Both papers price assets in a New Keynesian framework and attempt to explain changes in asset-price behavior through changes in the structural environment, as we do. However, neither focuses on monetary policy as a source of risk.

The paper proceeds as follows. In Section 2, we present our empirical evidence measuring differences in asset-price volatility across FOMC blackout and non-blackout periods and responses to MPR shocks on FOMC anouncement days. In Section 3, we explain our basic intuition by showing how risk premia on stocks and bonds should be related to MPR in a two-period linear-Gaussian economy, where simple closed-form solutions are available, and how these relationships compare to the effects of other kinds of risk. In Section 4 we develop our quantitative, nonlinear NK model and explain the solution method and parameterization. Section 5 presents the baseline results, calibrated our empirical evidence. Section 6 discusses the application to the experience of 2021 - 2023, and Section 7 concludes.

2 Quantifying Monetary-Policy Risk

2.1 A Simple Framework

We define monetary-policy risk as uncertainty about the policy rate, conditional on the state of the economy. To fix ideas, let i_t be the interest rate in period t and $\mathbf{x}_t$ be a vector representing the economic state. Suppose the joint evolution of monetary policy and the economy is described by the system

$$i_t = \boldsymbol{\phi}\mathbf{x}_t + v_t \tag{1}$$

$$\mathbf{x}_t = \mathbf{k}v_t + \mathbf{A}\epsilon_{x,t} \tag{2}$$

where v_t and $\epsilon_{x,t}$ are mean-zero stochastic processes that are exogenous with respect to $\mathbf{x}_t$ and i_t at all leads and lags. Following other work, we associate the interest rate with a short-term nominal bond yield, rather than the overnight interest rate, to allow expectations about future monetary policy to matter. v_t may thus be thought of as

reflecting central-bank forward guidance about the near-term path of policy as well as actual policy surprises in overnight rates. For expositional simplicity, we suppress intercept terms, abstract from possible inertia in the policy rule, and ignore the nominal lower bound on interest rates. These features, which we will include in our quantitative model below, do not affect the qualitative points we wish to make in this section.

In this framework, MPR at horizon h is given by

$$\text{var}_t[i_{t+h}|x_{t+h}] = \text{var}_t[v_{t+h}] \tag{3}$$

where var_t indicates uncertainty conditional on information at time t. We follow the long empirical and theoretical literatures that treat monetary policy as having such an exogenous stochastic component, which may arise from a variety of sources. The simplest case is the possibility of random departures from a central bank's fixed policy rule, as in the standard monetary-policy shocks that appear in NK models such as Rotemberg and Woodford (1997). Such shocks are isomorphic to transitory changes in the intercept of the rule—specifically, to possible variation in the central bank's inflation target. They may also be thought of as approximating, to a first order, changes in the parameters of the rule itself. For example, one interpretation of the events of 2022 was that the public became less sure about how aggressively the Fed would respond to inflation.[5] Another possibility is that monetary-policy "shocks" may also reflect differences between market and central-bank beliefs about the economy, as in Caballero and Simsek (2022). If so, then MPR may rise during periods of higher disagreement about macroeconomic fundamentals.

[5]Bauer, Pflueger, and Sunderam (2024) and Bocola et al. (2024) have recently presented evidence of medium-frequency shifts in coefficients of the FOMC's reaction function that could motivate such uncertainty. Expanding the model above to allow for stochastic parameters $\boldsymbol{\phi}_t$ in a linear approximation around the steady state produces $\text{var}_t[i_{t+h}|x_{t+h}] = \text{var}_t[\boldsymbol{\phi}_{t+h}] + \text{var}_t[v_{t+h}]$. Thus, an increase in MPR as we have defined it is observationally equivalent, to a first-order, to an increase in uncertainty about ϕ.

We remain agnostic about the microfoundations of v_t, but we assume that this component of the policy rule is stationary and potentially heteroskedastic. To be concrete, we specify

$$v_t = \rho_v v_{t-1} + \zeta_{t-1}\epsilon_{v,t} \tag{4}$$

where $\epsilon_{v,t} \sim Niid(0,1)$, and

$$\zeta_t = (1-\rho_\zeta)\bar{\zeta} + \rho_\zeta \zeta_{t-1} + \eta_t \tag{5}$$

with $\text{var}[\eta_t] = \omega^2$. With this structure, in period t, MPR at horizon $h = 1$ is simply equal to ζ_t^2, and MPR at all other horizons is proportional to this value.[6]

Let p_t be the log price of a financial asset. Assume that all such asset prices are functions of the economic and policy state variables:

$$p_t = f(v_t, \epsilon_{x,t}, \zeta_t) \tag{6}$$

Within this framework, we ask two questions. First, what is the overall contribution of MPR and its volatility to the variation of asset prices, i.e., what is the value of $\text{var}[p_t|\epsilon_{x,t} = 0]$? Second, what is the reaction of asset prices to a shock to MPR, $\partial f/\partial \zeta_t$?

2.2 Measuring the Contribution of MPR to Return Variation

To measure the overall contribution of MPR to asset-price volatility, we compare days when monetary-policy and MPR shocks are shut down (i.e., when $\epsilon_{v,t} = \eta_t = 0$) to days when these shocks are operative. Specifically, we exploit FOMC "blackout periods" to provide a measure of the background volatility in asset prices that is not related to

[6]For the moment, we ignore the positivity constraint on ζ_t, assuming that $\bar{\zeta}$ is far enough above zero that the process is essentially linear; this assumption will be relaxed in Section 4.

news about monetary policy. Blackout periods run from ten days before each FOMC announcement until one day after, and during these times Fed officials and staff refrain from communication with the public about matters having to do with monetary policy. Thus, variation in interest rates—and in other asset prices—during these periods must come entirely from news about shifting macroeconomic and financial conditions, rather than from news about the policy rule, possible deviations from the rule, or fluctuations in uncertainty about the rule. Comparing this volatility to the volatility on non-blackout days allows us to back out the contribution of monetary factors to the total variation of asset prices.[7]

Specifically, we can show

Proposition 1. In the economy described in equations (1) through (6), if ρ_ζ and ρ_v are close to 1, then

$$\underbrace{\text{var}[\Delta p_t | \epsilon_{x,t} = 0]}_{\text{contribution of MPR}} \approx \underbrace{\text{var}[\Delta p_t]}_{\text{non-blackout days}} - \underbrace{\text{var}[\Delta p_t | \epsilon_{v,t} = \eta_t = 0]}_{\text{blackout days}} \tag{7}$$

Proof. See Appendix A.

The quantity on the left-hand side of (7) is the measure we are interested in: the volatility of asset prices when all sources of risk other than monetary policy have been shut down. The assumption that ρ_ζ and ρ_m are close to 1 is likely to hold in the daily data we work with. Indeed, when we calibrate to match the data in Section 4, we find values of both of these parameters that are equivalent to more than 0.99 at a daily frequency. To the extent that the true values of ρ_ζ and ρ_m are less than one, our calculation will understate the size of MPR, so in this sense the assumption is

[7] The Fed's communications policy is available at https://www.federalreserve.gov/monetarypolicy/files/FOMC_ExtCommunicationStaff.pdf. If, despite the policy, some communication about monetary policy still takes place during blackout dates, our measurement provides a lower bound on the contribution of MPR.

conservative.

In the top panel of Table 1, we report the daily standard deviation of various asset prices and related financial variables on blackout days and non-blackout days, from 1995 through 2024.[8] We include FOMC announcement days in the "non-blackout" category, since they involve important communication to the public, but we also report those days separately for comparison. The table shows that, across all assets, there is significantly more volatility outside of blackout days than during blackout days. Although FOMC announcement days are particularly volatile, non-blackout days *other* than FOMC days also exhibit uniformly more asset-price volatility than blackout days do.

In the final column of the table we report the amount of volatility of each asset that is due to MPR. Specifically, using equation (7) we calculate $\sqrt{51 \times \text{var}[\Delta p_t | \epsilon_{x,t} = 0]}$, where the multiplier 51 scales the daily volatility to a quarterly frequency using the average number of non-blackout business days in each quarter (and approximating the asset prices as random walks over these short horizons). These results show that volatility associated with FOMC communications injects a substantial amount of risk into asset prices. For example, it contributes 3.0 percentage points (compared to a total of 8.5%) to the quarterly standard deviation of stock prices and 17 basis points (compared to a total of 41) to the quarterly standard deviation of the 10-year nominal yield. In terms of the stylized model above, these contributions reflect both the effects of shocks to monetary policy ($\epsilon_{v,t}$) and shocks to MPR itself (η_t).[9]

We also break out the results into periods at the ELB and away from the ELB, as shown in panels B and C. The contribution of MPR to stock-market volatility and to the volatility of the VIX are calculated to be somewhat smaller at the ELB, while the

[8] We include data from Treasury Inflation-Protected Securities (TIPS) only beginning in 2003 because market liquidity and data reporting were imperfect prior to that time.

[9] These results are not driven by outliers associated with extreme financial volatility or monetary-policy activism. In Appendix B, we report a robustness check in which we drop all observations between September 2008 and March 2009 and from March 2020. The calculations are quantitatively similar.

contribution to the volatility of real yields is slightly higher. On the whole, however, there are not large differences in asset-return volatilities on either blackout or non-blackout days across the two sub-samples.

2.3 Measuring the Effects of MPR Shocks

Our second empirical question is: what is the effect of a shock to MPR, η_t, on asset prices? Here, we follow Bauer, Lakdawala, and Mueller (2022), who conducted event studies to measure the reaction of financial-market variables to changes in near-term interest rates and interest-rate uncertainty on days of FOMC announcements, over the period 1994 – 2019. Under the assumption that macroeconomic news is negligible or uncorrelated with monetary-policy news on these days, regression coefficients of asset prices on measures of the policy stance and its uncertainty identify the effects of these variables on the asset price itself. This is because, in our notation, if there are no $\epsilon_{x,t}$ shocks, equations (1), (4), and (5) imply

$$\Delta i_t \approx \zeta_{t-1}\epsilon_{v,t} \tag{8}$$

$$\Delta \text{var}_t[i_{t+1}] \approx \eta_t \tag{9}$$

where, again, the equalities are exact when there is no predictability in returns (i.e., $\rho_\zeta = 1$ and $\rho_m = 1$—a close approximation at the daily frequency). Thus, a regression of Δp_t on Δi_t and $\Delta \text{var}_t[i_{t+1}]$, if feasible, would recover the values of $\partial f/\partial m_t$ and $\partial f/\partial \zeta_t$ local to the sample mean.

Following the other recent event-study literature, Bauer et al. (2022) use one-year interest rates, rather than the overnight federal funds rate, to capture the policy stance. This is appropriate in an environment where strong policy signals are sent in advance of movements in the actual policy rate, as was the case for most of the period covered

by the data. As the analogue of $\text{var}_t[i_{t+1}]$, they use the first principal component of implied volatilities of short-term interest rates up to one year ahead—which they refer to as "short-rate uncertainty"—as calculated from eurodollar options. We follow the same approach.[10] We also expand the regressions to examine some variables that Bauer et al. did not consider. Specifically, we include the five-year TIPS yield and the MOVE index of implied Treasury-yield volatility, which we collect from Bloomberg. All data cover the same time period as in the blackout exercise above.

The results for the full sample appear in Table 2.A. With respect to conventional monetary-policy shocks, we confirm other studies, such as Bernanke and Kuttner (2004), Gurkaynak et al. (2005), Hanson and Stein (2015), and Bauer et al. (2022) that document strong responses to such shocks across both equities and show that these results continue hold quantitatively even when changes in MPR are taken into account. For example, a 100 bp increase in the one-year policy rate raises the 5-year nominal yield by 40 bp and causes a 2.8% decline in the stock market. On the other hand, monetary-policy shocks do not have significant effects on implied volatilities of equities or interest rates.

Meanwhile, an increase in MPR, controlling for standard shocks to the level of the monetary-policy stance, is associated with a statistically significant fall in stock prices, increases in both real and nominal bond yields, and increases in implied volatility of both equities and longer-term Treasuries. Inflation compensation—the difference between the response of nominal and real yields—also declines in response to such shocks at both the five- and ten-year horizon. The magnitudes are economically large. For example, an increase in MPR that causes short-rate uncertainty to rise by 10 bp at the one-year horizon results in an 8.7 bp increase in the ten-year yield and a 0.85% decline in the S&P500. The fact that both the coefficients on TIPS yields are large

[10]We obtain the Bauer et al. data from Michael Bauer's website updated through 2024. https://www.michaeldbauer.com.

and positive is strongly suggestive of an increase in real term premia in response to MPR shocks, since it is unlikely that monetary policy would be expected to tighten in response to these shocks and investors would have to expect such a tightening to be very persistent to explain a large rise in the 5-year-ahead forward real rate.[11]

We also extend the Bauer et al. (2022) analysis by examining differences across ELB and non-ELB periods. Specifically, we interact the monetary-policy and short-rate-uncertainty shocks with an indicator variable that takes a value of 1 on dates when the federal funds rate was at the ELB. Panels B and C of Table 2 present the results of the regressions with these interaction terms. We find essentially no differences in the effects of conventional monetary-policy shocks at the ELB. In contrast, we find that the ELB substantially enhances the effects of MPR on asset prices. The effect of changes in MPR on the ten-year nominal yield, for example, are more than twice as large for the ELB observations than in the rest of the sample, and for the stock market the increase in the coefficient is nearly five-fold. With the exception of the MOVE, the increases in magnitude at the ELB are statistically significant at at least the 5% level.

3 Intuition under Log Normality

In our empirical exercises above, we demonstrated (1) that uncertainty about monetary policy, conditional on the state of the economy, contributes importantly to return volatility across asset classes; and (2) that positive shocks to this kind of uncertainty cause significant price declines across asset classes. Our structural model of Section 4 will match these qualitative results, as well as most of the quantitative estimates in Tables 1 and 2. Because that model does not have a closed-form solution, however, we first explain in a simpler context how we would expect MPR to translate into asset

[11] As above, the results are quantitatively similar if we drop observations during the financial crises in 2008 - 09 and the early part of the pandemic; see Appendix B.

prices and how those effects compare to the effects of other possible types of risk in the economy.

3.1 Reduced-Form Assumptions

Building on the linear macroeconomic framework developed above, let the economic state be $\mathbf{x}_t = [y_t, \pi_t]$, where y_t is the output gap and π_t is log of the gross change in the price level between periods t and $t-1$; let ϕ_y and ϕ_π be the coefficients on output and inflation in the monetary-policy rule; let $\mathbf{k} = [k_y, k_\pi]$ be the reduced-form macroeconomic effects of the monetary-policy shock; and let $\mathbf{A} = \left(\begin{smallmatrix} a_y^d & a_y^s \\ a_\pi^d & a_\pi^s \end{smallmatrix}\right)$ be the reduced-form effects of macroeconomic "demand" and "supply" shocks $\boldsymbol{\epsilon_t} = [\epsilon_{d,t}, \epsilon_{s,t}]$. We assume that $\epsilon_{d,t}$,$\epsilon_{s,t}$, and $\epsilon_{v,t}$ are mean zero, normally distributed, independent, and serially uncorrelated. We assume constant variances σ_d^2 and σ_s^2 for the macroeconomic shocks. As discussed above, the variance of the monetary-policy shock varies over time as $\text{var}_t[\epsilon_{v,t+1}] = \zeta_t^2$. We further assume $\phi_\pi > 1$ and $\phi_y > 0$, consistent with a typical policy rule that satisfies the Taylor principle. Finally, we assume that the reduced-form coefficient values determining the macroeconomic variables are such that the three shocks have the typical properties of aggregate-demand, aggregate-supply, and monetary-policy shocks in models with monetary non-neutrality. Specifically, we assume $a_\pi^s < 0$, $a_y^s > 0$, $a_\pi^d > 0$, and $a_y^d > 0$, and we assume that $\mathbf{k}$ satisfies $k_\pi < 0$, $k_y < 0$, and $\boldsymbol{\phi}'\mathbf{k} > -1$. This last restriction ensures that positive monetary-policy shocks generate a contemporaneous rise in both real and nominal short-term interest rates.

In order to be more explicit about the pricing function in (6), we now require the one-period real stochastic discount factor (SDF) $M_{t+1} = \exp(m_{t+1})$ in period t. An absence of equilibrium arbitrage opportunities and non-pecuniary motives for holding financial assets implies that M_{t+1} prices all real assets in the economy. In this section,

for analytical tractability, we assume that the log SDF, reflecting investors' relative marginal utility in successive periods, is given by

$$m_t = -r_{t-1} + \mu^d \epsilon_{d,t} + \mu^s \epsilon_{s,t} + \mu^v \zeta_{t-1} \epsilon_{v,t} - \frac{\sigma_m^2}{2} \tag{10}$$

where r_t is the real one-period interest rate and σ_m^2 is the (constant) conditional variance of m_t. We assume that the effects of each of the shocks on real activity is directionally the opposite as its effect on real output, so that $\mu^v > 0$, $\mu^d < 0$, and $\mu^s < 0$. We summarize the coefficient sign assumptions in Table 3.

3.2 Asset Prices in the Log-Normal Model

In this setting, the one-period interest rate carries an inflation risk premium (IRP) equal to $\text{cov}_t[m_{t+1}, \pi_{t+1}]$. (See Appendix A for derivations of standard asset pricing equations here.) This premium represents the compensation that holders of the one-period bond demand for uncertainty about their real payoff one period hence. The IRP is higher when inflation tends to occur in bad states of the world, because in these cases the real payoffs on nominal bonds are lower in bad times.[12] Under the linearity and homoskedasticity assumptions of this section, this term, like all risk premia, is constant. In the nonlinear model presented below, it will fluctuate with the economic state.

In particular, we can expand the one-period inflation-risk premium as follows:

$$IRP^{(1)} = a_\pi^s \mu^s \sigma_s^2 + a_\pi^d \mu^d \sigma_d^2 + \kappa_\pi \mu^v \zeta_t^2 \tag{11}$$

Given the signs of the parameters assumed in Table 3, it follows that monetary-policy

[12]Note that, because of this term, the Fisher equation does not generally hold. Again, this is a standard result under no-arbitrage.

risk, as captured by the last of these three additive terms, contributes negatively to this premium. Thus, when MPR (ζ_t^2) increases, the effect on the inflation-risk premium is negative. Intuitively, monetary-policy shocks move inflation and output (marginal utility) in the same (opposite) direction. When such shocks are more prevalent—i.e., MPR increases—the inflation risk of nominal bonds falls. The effect of uncertainty about demand shocks is the same, as demand shocks themselves send output and inflation in the same direction. In contrast, the impact of uncertainty about supply shocks is positive.

The real one-period bond has no risk premium, but the real two-period bond yield carries a real term premium (RTP), which, as Appendix A shows, is given by

$$\begin{aligned} RTP_t^{(2)} \equiv \mathrm{cov}_t[m_{t+1}, r_{t+1}] = &\left[\phi_y a_y^s + (\phi_\pi - 1)a_\pi^s\right] \mu^s \sigma_s^2 \\ &+ \left[\phi_y a_y^d + (\phi_\pi - 1)a_\pi^d\right] \mu^d \sigma_d^2 + (1 + \boldsymbol{\phi}'\mathbf{k} - \kappa_\pi)\, \mu^v \zeta_t^2. \end{aligned} \tag{12}$$

It reflects compensation for comovement between the SDF and the payoffs on real bonds (which are inversely related to the $t+1$ real short rate). Shocks that move payoffs on real bonds and marginal utility in opposite directions increase the real term premium. In particular, from (12), a higher level of MPR results in a higher real term premium, since, under our assumptions, $(1 + \boldsymbol{\phi}'\mathbf{k} - \kappa_\pi)\, \mu^v$ is strictly positive. The reason is straightforward. In economies with nominal rigidities, monetary-policy shocks move payoffs on real bonds and marginal utility in opposite directions. When such shocks are more prevalent—i.e., MPR increases—the real risk of real bonds rises. Meanwhile, a higher level of demand-side risk σ_d^2 lowers the real term premium (bonds are a hedge to bad demand shocks). The observation that demand-side risk implies negative real term premia has presented a puzzle when it comes to explaining empirical real term premia that appear to be positive on average.[13] Moreover, the theoretical

[13]E.g., Li and Palomino (2014). As noted above, D'Amico, Kim, and Wei (2018) estimate a 10-year

effects of supply-side risk on the real term premium are ambiguous; with the additional restriction that real rates must rise in response to a supply shock ($\phi_y a_y^s + \phi_\pi a_\pi^s > 1$), supply side uncertainty raises the RTP. However, models that explicitly incorporate this channel (e.g., Swanson, 2021) produce real term premia that are quantitatively small. MPR presents a possible resolution to this puzzle. As we will see below, our quantitative results suggest that its contribution is substantial.

The two-period nominal term premium includes the two-period real term premium and the one-period inflation-risk premium, and it also contains additional terms reflecting the covariance between inflation and the SDF across the second period.

$$\begin{aligned} NTP_t^{(2)} &\equiv RTP_t^{(2)} \\ &\quad + \underbrace{IRP_t^{(1)} + \text{cov}_t[\pi_{t+1}, m_{t+2}] + \text{cov}_t[\pi_{t+2}, m_{t+1}] + \text{cov}_t[\pi_{t+2}, m_{t+2}]}_{IRP_t^{(2)}}. \end{aligned} \tag{13}$$

What matters for the IRP on two-period bonds is the covariance between the cumulative values of inflation and the SDF, which involves their dynamic cross-covariances. Even in our simple model, these terms are somewhat tedious to work out algebraically. But, for similar reasons to those we discussed in the case of the one-period inflation-risk premium, those calculations show that the effect of MPR on $IRP^{(2)}$ are negative. It turns out that the effects of MPR on the RTP are always greater than those on the IRP, so that the nominal term premium rises with greater MPR. (See Proposition 2 below.)

Turning to equities, consider the 2-period equity claim which pays a dividend equal

real term premium of over 60 basis points over our sample period. A model free statistic is that the 10-year TIPS yield exceeded the 5-year TIPS yield by 42 on average between 2003 and 2025.

to output $\exp[y_{t+1}]$. The expected log nominal return on this asset is given by

$$\mathrm{E}_t\left[r^{eq}_{t+1}+\pi_{t+1}\right] = i_t \underbrace{-\mathrm{cov}_t[m_{t+1},y_{t+1}]}_{ERP_t} + \underbrace{\mathrm{cov}_t[\pi_{t+1},y_{t+1}]-\frac{1}{2}\mathrm{var}_t[y_{t+1}]}_{\text{Jensen}} \quad (14)$$

The first term is nominal risk-free rate, and the second term, reflecting the covariance between dividends and the SDF, is the equity risk premium. In our setting, the ERP is:

$$ERP = -\mathrm{cov}_t[m_{t+1},y_{t+1}] = -a^s_y\mu^s - a^d_y\mu^d - \kappa_y\mu^v\zeta_t^2 \quad (15)$$

Monetary-policy shocks cause nominal dividends to fall in bad states of the world, so an increase in MPR pushes the equity premium higher.[14]

The following proposition summarizes the results of this section:

Proposition 2. In the log-linear Gaussian model described above, an increase in monetary-policy risk ζ_t results in:

- higher variance of real and nominal interest rates and bond yields;
- higher variance of returns on real bonds, nominal bonds, and equities;
- a higher level of real and nominal term premia;
- a lower level of inflation risk premia;
- a higher level of the equity premium;

Proof. See Appendix A.3.

The effects of MPR on risk premia are summarized in Table 4. Although we will not focus on the effects of changes in aggregate-demand and aggregate-supply risk,

[14]The last two terms in (14) reflect an additional effect stemming from the convexity of the log return. The net effect of MPR on these terms depends on monetary policy shocks' relative impact on inflation and dividends.

these can be calculated in the same manner and are also reported for comparison. It is instructive to note the ways in which different structural sources of risk have similar effects and the ways in which they differ. For example, although all three sources of risk increase the equity premium, they have different directional effects on the components of the nominal term premium. Supply-side risk, like monetary-policy risk, makes nominal and real bonds riskier, since supply shocks cause interest rates to rise in bad states of the world. Demand shocks, in contrast, are associated with *lower* short-term interest rates in bad states, and thus a higher variance of demand shocks makes bonds more of a hedge. Thus, supply and monetary-policy risk lead to higher real and nominal term premia, while demand risk reduces them. On the other hand, supply and monetary-policy risk differ in their implications for the inflation-risk premium. This is because supply shocks move inflation and real activity in opposite directions, while monetary-policy shocks move them in the same direction.

The results of this section have focused on the effects of MPR on risk premia. But asset prices are also affected by the expected path of risk-free rates and, in the case of equities, expected dividends. In general, risk shocks reduce economic activity, implying that both risk-free rates and dividends will typically fall in the face of MPR shocks. In simple New Keynesian models, like the one we present in the next section, this occurs via an increase in precautionary savings, but in reality there are likely a variety of channels through which increases in macroeconomic uncertainty have economically depressing and disinflationary effects (see Bloom (2014)). All else equal, such outcomes induce an endogenous monetary-policy easing. In other words, increases in risk behave like negative aggregate demand shocks with respect to macroeconomic variables (Leduc and Liu (2016)). As a result, the overall effects on stock returns and on the real and nominal yield curves are complex and can only be determined in the context of a fully specified general-equilibrium model. We turn to such a model next.

4 Monetary-Policy Risk in a New Keynesian Model

We now develop a formal structural model to micro-found and quantify the intuitive effects discussed above. For monetary policy and its risk to be theoretically interesting, we require nominal rigidities, so we work within the New Keynesian paradigm. In order to isolate the effects we are interested in, we keep the model as simple as possible. However, we add three somewhat non-standard features that help us understand the effects of MPR on asset prices. First, we endow households with recursive preferences over consumption (Epstein and Zin (1991)), which helps to deliver realistic behavior of risk premia. Second, we introduce stochastic volatility into the shocks in the monetary-policy rule. This is how we model transitory fluctuations in MPR. Third, we allow for the effective lower bound on nominal rates, a feature that has been a significant constraint on policy in recent years. As suggested above, including the ELB is important for explaining changes in asset-price behavior over time, as the proximity of the ELB changes the quantity and nature of interest-rate risk and thereby affects term premia (King (2019), Gourio and Ngo (2020)).

4.1 Model specification

4.1.1 Monetary policy and MPR

Expanding equation (1), we assume that the central bank follows an inertial policy rule for the short-term interest rate, subject to an effective lower bound. We model the ELB using a shadow-rate process, as in Fernandez-Villaverde et al. (2015); Gourio and Ngo (2020), and others:

$$\hat{i}_t = (1-\varphi_i)\left[\pi^* + r^* + \phi_y \widetilde{y}_t + \phi_\pi(\pi_t - \pi^*)\right] + \varphi_i \hat{i}_{t-1} + v_t \tag{16}$$

$$i_t = \max[\hat{i}_t, 0] \tag{17}$$

where π^* is the long-run inflation target, r^* is the steady-state real interest rate, $\widetilde{y}_t$ is the log deviation of output from its steady-state value, and v_t is the monetary-policy shock. We continue to assume that v_t follows the dynamics given in equation (4) and that its conditional standard deviation, given by the state variable ζ_t, follows the AR(1) dynamics in (5). However, in order to rule out negative variance, we now assume that the distribution of η_t is subject to truncation below at $-\mathrm{E}_{t-1}[\zeta_t]$.[15] Because, under our calibration, ζ_t is typically far from zero, this process is nearly linear and homoskedastic over most of the relevant range, including near the steady state. We specifically choose to model the stochastic volatility term as a linear, rather than log-linear or square-root, process to ensure that the conditional variance of MPR $\mathrm{var}_t\left[\zeta_{t+1}\right]$ is nearly constant. Under a log process, this variance increases with η_t. Since, as we argued above, MPR shocks act similarly to demand shocks, increases in their conditional variance have qualitatively different effects on asset prices than increases in the conditional variance of v_t, and nonlinear processes for ζ_t confound these two effects. The linear process, in contrast, allows us to isolate the effects of $\mathrm{var}_t[v_{t+1}]$ that we are interested in.[16]

4.1.2 Households

We want to exploit the attractive asset-pricing properties of recursive preferences over consumption (C_t) within the context of a New Keynesian model where households also receive disutility from labor (H_t). The question of how best to extend recursive preferences to include leisure is not settled in the literature, and the few other papers that have worked with such models have done it in different ways.[17] Our approach is

[15] Specifically, the PDF of η_t is $\frac{1}{\sqrt{2\pi}\omega^{3/2}}\exp\left[-\frac{\eta_t}{2\omega^2}\right]\left(1-\mathrm{erf}\left[\frac{(1-\rho_\zeta)\bar{\zeta}+\rho_\zeta\zeta_{t-1}}{\sqrt{2}}\right]\right)$.

[16] The linear specification follows Mumtaz and Zanetti (2013) and Basu and Bundick (2017), although, because those papers solve by perturbation they sidestep the problem of the non-negativity constraint.

[17] See, for example, the various functional forms adopted by Dew-Becker (2014), Basu and Bundick (2017), and Swanson (2021).

to assume that households' lifetime utility is given by preferences of the following form:

$$U_t = \left[(1-\beta)(C_t^{\rho} - \theta_t H_t^{\chi\rho}) + \beta\left(\mathscr{R}_t\left(V_{t+1}^{C}\right)^{\rho} + \mathscr{R}_t\left(V_{t+1}^{H}\right)^{\rho}\right)\right]^{1/\rho} \tag{18}$$

where θ_t is an exogenous process governing the consumption-leisure trade-off, the certainty-equivalence function $\mathscr{R}_t(.)$ is defined by

$$\mathscr{R}_t(X_{t+1}) \equiv E_t[X_{t+1}^{\alpha}]^{1/\alpha}$$

for any random variable X_{t+1}, and the continuation values of consumption and leisure are given by the recursions

$$V_t^C = \left[(1-\beta)C_t^{\rho} + \beta\mathscr{R}_t\left(V_{t+1}^{C}\right)^{\rho}\right]^{1/\rho}$$

$$V_t^H = \left[(1-\beta)\theta_t H_t^{\chi\rho} + \beta\mathscr{R}_t\left(V_{t+1}^{H}\right)^{\rho}\right]^{1/\rho}$$

This specification nests both Epstein-Zin preferences over consumption (if $\theta_t = 0$ for all t) and the additively separable preferences over consumption and leisure that are standard in the NK literature (if $\alpha = \rho$). It has the convenient feature that the intra-temporal optimality condition will not involve the recursive terms, preserving the usual tractability of general-equilibrium business-cycle models, but that the *inter*-temporal optimality condition will reproduce the Epstein-Zin discount factor familiar from consumption-based asset pricing.

Letting P_t be the price of the consumption good and W_t be the nominal wage, the household maximizes lifetime utility subject to the flow budget constraint:

$$P_t C_t + P_t^{\$(1)} B_t \leq B_{t-1} + W_t H_t + \mathscr{P}_t \tag{19}$$

where $\mathscr{P}_t$ is firm profits. Since the model will feature a trend growth rate g in log real wages and log consumption, to keep hours stationary, we assume that θ_t grows at a constant rate $-\rho g$, as in Rudebusch and Swanson (2012).

4.1.3 Firms

The production side of our economy is completely standard. Firms produce differentiated goods, indexed by i, according to the Cobb-Douglas production function

$$y(i)_t = a_t^* + \psi \log H(i)_t \tag{20}$$

where the steady-state level of log productivity a_t^* follows deterministic growth

$$a_t^* = a_{t-1}^* + g$$

Firms face iso-elastic demand from households and are subject to Calvo pricing. Aggregate output is given by the Dixit-Stiglitz aggregate of $e^{y(i)}$.

4.2 Equilibrium and solution

Appendix C shows that the optimality conditions for the household's problem can be written as

$$\chi \theta_t H_t^{\chi\rho-1} C_t^{1-\rho} = \frac{W_t}{P_t} \tag{21}$$

$$\widetilde{C}_t = \exp[g] \left(\beta \exp[i_t] E_t \left[\frac{\widetilde{C}_{t+1}^{\rho-1}}{\Pi_{t+1}} \Big(\frac{\widetilde{V}_{t+1}^C}{\mathscr{R}_t(\widetilde{V}_{t+1}^C)} \Big)^{\alpha-\rho} \right] \right)^{\frac{1}{\rho-1}} \tag{22}$$

$$\widetilde{V}_t^C = \left[(1 - \beta \exp[g\rho]) \widetilde{C}_t^\rho + \beta \exp[g\rho] \mathscr{R}_t \left(\widetilde{V}_{t+1}^C \right)^\rho \right]^{1/\rho} \tag{23}$$

where tildes denote percentage deviations from trend. After log-linearizing, the firm's problem, combined with the labor-supply curve (21), results in the New Keynesian Phillips Curve:

$$\pi_t = \pi^* + \beta \exp[g(\rho - 1)] E_t[\pi_{t+1} - \pi^*] - \nu \widetilde{a}_t + \kappa \widetilde{y}_t \tag{24}$$

where ν and κ are reduced-form parameters that take non-negative values. Note that, in order to obtain meaningful risk premia, we do not linearize the Euler equation.

A solution to the model consists of values of C_t, H_t, W_t, P_t, and $P_t^{\$(1)}$ such that equations (21) through (24) are satisfied and the goods, bond, and labor markets clear ($C_t = \exp y_t$, $B_t = 0$, and $\int H(i)_t = H_t$) for all t.

4.3 Asset Prices

The log SDF in this model is given by

$$m_{t+1} = \log \beta + (\rho - 1)(g + \widetilde{c}_{t+1} - \widetilde{c}_t) + (\alpha - \rho)(\log \widetilde{V}_{t+1}^C - \log \mathscr{R}_t(\widetilde{V}_{t+1}^C)) \tag{25}$$

and the prices of nominal and real bonds of maturity N are given by

$$P_t^{\$(N)} = E_t \left[\frac{M_{t+1}}{\Pi_{t+1}} P_{t+1}^{\$(N-1)} \right] \tag{26}$$

$$P_t^{(N)} = E_t \left[M_{t+1} P_{t+1}^{(N-1)} \right] \tag{27}$$

The yield on a nominal bond is defined as

$$y_t^{\$(N)} \equiv -\frac{1}{N} \log P_t^{\$(N)} \tag{28}$$

and similarly for real yields. "Inflation compensation" is the difference between nominal yields and real yields. One-period yields on real and nominal bonds are the short-term interest rates, $r_t = y_t^{(1)}$ and $i_t = y_t^{\$(1)}$.

The real term premium is the difference between the yield on an inflation-indexed bond and the value that the yield would take if agents were risk-neutral. Under risk neutrality, assets are priced as if M_{t+n}, for all n, is always equal to its time-t expectation with certainty. In this case, the term structure of interest rates is given by the geometric average of expected future short-term rates. In Appendix A, we define a geometric expectation operator ($\mathscr{E}_t^{(N)}[\cdot]$), which subsumes Jensen's inequality terms into physical expectations of future prices and cash flows and leave risk premia as clean measures of deviations from risk neutrality. Thus, the real term premium (RTP) is

$$RTP_t^{(N)} = y_t^{(N)} - \mathscr{E}_t^{(N)}[r_t] \tag{29}$$

The inflation risk premium (IRP), representing the extra yield that nominal bonds must pay over real bonds for bearing the risk of nominal payoffs, is defined as inflation compensation less expected inflation:

$$IRP_t^{(N)} = y_t^{\$(N)} - y_t^{(N)} - \mathscr{E}_t^{(N)}[\pi_t] \tag{30}$$

We then define the nominal term premium (NTP) as

$$\begin{aligned} NTP_t^{(N)} &= RTP_t^{(N)} + IRP_t^{(N)} && (31) \\ &= y_t^{\$(N)} - \left(\mathscr{E}_t^{(N)}[r_t] + \mathscr{E}_t^{(N)}[\pi_t]\right) && (32) \end{aligned}$$

Note that, although we have defined the RTP in the usual intuitive way, as the difference between real yields and real rate expectations, the NTP is not defined analogously

(since $\mathscr{E}_t^N[i_t] \neq \mathscr{E}_t^N[r_t] + \mathscr{E}_t^N[\pi_t]$). This asymmetry is necessary to ensure that the IRP and RTP add up to the NTP. Although this definitional choice is not important for our results, one implication is that short-term nominal bonds carry a term premium, consisting entirely of $IRP_t^{(1)}$. In other words, as in the model of Section 3, the Fisher equation does not generally hold because even one-period nominal bonds bear some amount of inflation risk, which requires compensation. Again, this is a standard no-arbitrage result.

The nominal return on any bond is its real return plus inflation. One-period log real returns on nominal bonds and real bonds are thus as follows:

$$r_{t+1}^{\$(N)} = \log P_{t+1}^{\$(N-1)} - \log P_t^{\$(N)} - \pi_{t+1} \tag{33}$$

$$r_{t+1}^{(N)} = \log P_{t+1}^{(N-1)} - \log P_t^{(N)} \tag{34}$$

To price equities, first consider a hypothetical contract that pays a stream of cash flows equal to aggregate nominal output. The price of such a contract can be found recursively as

$$P_t^y = \mathrm{E}_t[\exp(m_{t+1} - \pi_{t+1})(P_{t+1}^y + \exp(y_{t+1} + \pi_{t+1}))] \tag{35}$$

and its log real returns are

$$r_{t+1}^y = \log\left[P_t^y + \exp(y_t + \pi_t)\right] - \log P_{t-1}^y - \pi_{t+1} \tag{36}$$

Following Campbell (1986), we model equity contracts as *levered* claims on output. To fund levered positions, firms are assumed to borrow a multiple $1/\delta$ of their equity value at the short-term nominal rate i_t, rolling over this debt every period. In this

case, Campbell shows that the real return on equities is given by

$$r_{t+1}^{eq} = \log\left[\frac{1}{\delta}\exp(r_t^y + \pi_t) - \frac{1-\delta}{\delta}\exp(i_{t-1})\right] - \pi_{t+1} \tag{37}$$

The equity-risk premium (ERP) is the expectation of the nominal equity return, less the value that this expectation would take under risk neutrality, implying

$$ERP_t = \mathscr{E}_t[r_{t+1}^{eq} + \pi_{t+1}] - i_t \tag{38}$$

Because of convexity terms, this is slightly different from the expected excess returns on equities, which are given by $\mathrm{E}_t\left[r_{t+1}^{eq} + \pi_{t+1}\right] - i_t$.[18]

4.4 Calibration and Solution

Table 5 presents the parameter values we use in our baseline model. Time is quarterly. We calibrate a subset of our parameters to standard values. Specifically, we set the quarterly rate of time preference β to 0.99, the intertemporal elasticity of substitution to 2, the labor share ψ to 0.35, and the slope of the Phillips curve to $0.05/4$.[19] We calibrate the response coefficients in the policy rule to the Taylor (1993) values of $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, and we set $\pi^* = 1.9/4$ and $r^* = 0$ to match the average values of inflation and short-term interest rates since the mid-1990s.[20]

The remaining values we calibrate to match the empirical results presented ear-

[18]Note that, while this is typically how excess returns are measured in the data, some theoretical treatments focus on what one might call the "real excess returns," $\mathrm{E}_t[r_{t+1}^{eq}] - r_t$. These are not generally the same because the Fisher equation does not hold. Unless otherwise noted, we work with the nominal returns.

[19]The frequency of Calvo price adjustments and the parameter governing the elasticity of labor supply (ξ) both enter into the Phillips Curve slope. Since we do not focus on any results concerning wages, these parameters enter nowhere else in our model. Thus, we calibrate the Phillips Curve slope directly, rather than taking a stand on the values of these two structural parameters.

[20]Given the other parameters, calibrating r^* is equivalent to calibrating the steady-state growth rate g.

lier. Specifically, we choose parameters that minimize the deviations from the implied quarterly volatilites due to MPR across all assets, shown in Table 1, and the responses of all asset prices to monetary-policy and MPR shocks, shown in Table 2. In doing these comparisons, we evaluate the model at the steady state and match to the empirical results in the samples that exclude periods at the ELB. Conditional on the other parameters, the variances of the monetary-policy and MPR shocks ζ and σ_v are identified from the differences in asset-price volatilities between blackout and non-blackout days reported in Table 1. In particular, these data lead us to calibrate a steady-state conditional standard deviation of monetary-policy shocks ζ of 89 basis points and a conditional standard-deviation of MPR shocks of 30 bp (with interest rates expressed as annual rates).[21] Meanwhile, the relative responses of real and nominal yields of different maturities to the two shocks largely pin down the persistence parameters ϕ_i, ρ_z, and ρ_v. All three of these take relatively modest values between 0.65 and 0.75 at a quarterly frequency, but in combination they will imply somewhat persistent dynamics for MPR shocks.

Finally, in order to match the empirical responses to monetary-policy and MPR shocks in Table 2, the model requires $\alpha = -118$ and $\delta = 0.79$. As is well known, matching equity and bond returns requires a high level of risk aversion, and indeed the principal advantage of Epstein-Zin preferences are that they allow us to set α to a high value while maintaining an empirically realistic IES. While $\alpha = -118$ is unrealistic at the level of an individual consumer, we view the representative-household preferences used here as an aggregate across many individuals and institutions in the economy, which need not preserve microeconomic parameter values. (Gourio and Ngo (2020)

[21] The value of ζ is somewhat high compared to the conditional variance of short-term interest rates in the data. However, shocks to monetary policy do not pass through one-to-one to short-term interest rates, because they have effects on output and inflation that negatively feed back through the policy rule. In addition, ζ needs to capture uncertainty about the entire path of policy, and, in the data, that uncertainty is considerably larger beyond the very short term. For example, 12-month option-implied uncertainty of the federal funds rate averages about 100 bp, a value that our model roughly matches.

make a similar point.)

Previous papers that have studied asset prices in DSGE models have often relied on higher-order perturbation methods to solve them.[22] Although this approach is relatively efficient and can give accurate approximations for macroeconomic dynamics near the steady state, it is not likely to have good properties for asset prices in the presence of significant nonlinearities. This is because asset prices can be sensitive to the magnitudes of outcomes with low unconditional probabilities, which are necessarily far away from the steady-state perturbation point. If the policy functions are not sufficiently smooth, polynomial approximations far from this point can be quite poor.[23] These problems may be compounded if the model involves nondifferentiable functions, such as those induced by the ELB. To avoid these issues, we rely on discretization and interpolation across the state space, a method that is computationally intensive but provides a globally accurate solution, even in the presence of occasionally binding constraints. Appendix D describes the numerical procedure in detail.

5 Model Results

5.1 Matching empirical moments

Table 6 shows how the model performs in matching the evidence on the contribution of MPR to asset-price volatility that we derived from blackout-period data in Table 1. Specifically, we calculate the unconditional standard deviations of various asset prices in the model. Since MPR and its volatility are the only sources of variation in the model, we can compare these numbers to the contributions of MPR and its volatility that we isolated in the data using the blackout-period evidence. Although

[22] Obtaining time-varying risk premia requires perturbation of at least the third order. Examples include Rudebusch and Swanson (2012), Swanson (2021), and Mumtaz and Zanetti (2013).

[23] See Aldrich and Kung (2019).

we calibrated the parameters of the model to roughly match these results, as well as those in Table 2, we have many more empirical moments than model parameters so a good overall fit is not guaranteed. Even so, the model is generally able to match the difference in variance between blackout and non-blackout days of real and nominal bond prices and their implied volatility. On the other hand, the model falls short in explaining the apparent contribution of MPR to stock-market volatility. In the data, we estimated that this contribution was about 3.4% per quarter, whereas the model generates only 1.9%. Similarly, the model predicts a lower volatility of the VIX index than we calculated in the data.

Table 7 shows how the model performs in matching the evidence on the responses of asset prices to both MPR shocks and monetary-policy shocks, based on the event studies around FOMC announcements that we presented in Table 2. In the model, we construct the responses to MPR shocks by introducing a shock of 1 bp η, starting from the steady state. Because the empirical regressions control for changes in one-year yields, we simultaneously introduce a shock ϵ_v that maintains the value of the one-year yield exactly at its initial value. To achieve comparability to the empirical exercise, we report the initial response of the asset price of interest to this combination of shocks, divided by the response of the four-quarter-ahead volatility of the short rate to the same combination of shocks. Similarly, when comparing to the empirical responses of asset-prices to monetary-policy shocks, we set ϵ_v to one basis point, starting from the steady state, and we simultaneously introduce a shock η that maintains the value of the one-year-ahead short-rate volatility exactly at its initial value. (Because there is not much effect of the rate path on rate volatility near the steady state, this latter adjustment is small.) To achieve comparability to the empirical exercise, we report the initial response of the asset price of interest to this combination of shocks, divided by the response of the one-year yield to the same combination of shocks. When

examining both MPR and monetary-policy shocks at the steady state, we compare to the empirical results in the sample away from the ELB. The case of shocks at the ELB will be examined below.

For both types of shocks, the model performs well in matching the regression results. For monetary-policy shocks, it matches every coefficient to within one standard error. For MPR shocks, all of the model-implied moments have the same sign as we find in the data, and the magnitudes of the responses are almost all within two standard errors of the regression point estimates. Quantitatively, the model does err somewhat to the downside when it comes to the responses of long-term nominal yields to MPR shocks and somewhat to the upside for the responses of the stock market. This likely reflects a tension between these two sets of empirical results in terms of their implications for the persistence of MPR shocks. Even so, given the relatively small number of free parameters, the model does remarkably well at explaining the the overall behavior of the data with respect to monetary and MPR shocks.

5.2 Contributions to risk premia

Table 8 reports unconditional means of the nominal ten-year term premium, inflation risk premium, real term premium, and equity premium and the contribution to these overall values that come from MPR and from the volatility of MPR. We measure contribution of the volatility of MPR shocks as the difference between the model with homoskedastic policy shocks and the baseline model that allows for stochastic variance.[24]

The existence of monetary policy shocks contributes 62 basis points to the average value of the ten-year nominal term premium, 97 basis points of which comes through

[24]In the homoskedastic model, ζ is adjusted such that the policy shocks have exactly the same unconditional variance as the shocks in the baseline model.

the real term premium. The average slope of the real yield curve is determined by real term premia. Thus, compensation for risk arsing from monetary policy is one potential explanation for why the real yield curve slopes upward on average, and this effect passes through to nominal yields as well. On the other hand, for reasons discussed above, MPR induces a negative inflation-risk premium of 32 basis points on average. Additionally, MPR drives a substantial portion of the equity premium, contributing a total of 160 basis points.

MPR shocks—that is, shocks to the conditional standard deviation of policy shocks—have only minor contributions to risk premia. Specifically, they contribute -2 bp to real term premia. A negative contribution occurs because risk shocks behave like demand shocks in the sense that they send bond returns higher in bad states of the world and therefore increase the hedging property of bonds. For similar reasons, they also make a small negative contribution to the average values of the inflation risk premium and a small positive contribution to the equity premium. (The variance of MPR also contributes to the existence of negative variance risk premia, not shown, though these effects are relatively modest.)

5.3 Effects of shocks at the steady state

As shown in Figure 2, monetary-policy shocks in our model have effects on the macroeconomy and asset prices that are fairly standard. When the economy is at the steady state, a 25 bp exogenous increase in the short-term interest rate reduces output by about 0.8% and inflation by about 0.2% on average over the subsequent year. On impact, it increases the 5-year nominal yield by 9 basis points, increases the 5-year real yield by 14 basis points (so, 5-year inflation compensation falls by 5 basis points), and causes stock returns to fall by about 1.1%. As we showed in Table 7, these asset-price responses are all quantitatively very similar to those estimated from the event-study

data. Near the steady state, monetary-policy shocks do not have appreciable effects on volatilities or risk premia.

Our main objects of interest are the responses to the MPR shocks. The top row of Figure 3 shows that a one-standard-deviation positive shock to MPR (an increase of 30 bp in the one-quarter-ahead standard deviation of the annualized short rate) causes the conditional(one-period-ahead) standard deviation of the nominal interest rate to increase by 18 basis points, with corresponding increases in the conditional standard deviations of output and inflation. As shown in the second row, this increase in uncertainty raises real risk premia for both bonds and equities, with the 5-year real term premium increasing 12 bp and the equity premium rising 25 bp. For reasons discussed above, the inflation risk premium declines slightly, but the magnitude is less than the increase in the real term premium, so that the nominal term premium (not shown separately) rises.

In addition to the effects on risk premia, the MPR shocks also have effects on risk-free rates and cash flows through macroeconomic channels. As shown in the third row of the figure, the MPR shock by itself reduces both output and inflation modestly in the short run. These effects are damped by the endogenous monetary-policy response—the shock triggers a short-run decline of about 20 bp in the policy rate. In some situations, it may be more realistic to assume that the central bank does not ease its current policy in response to uncertainty about its future policy. We can construct such a scenario by introducing a sequence of additional monetary-policy shocks to keep the policy rate unchanged over time. This scenario is depicted by the green lines.[25]

The final two rows of the figure show how these effects net out for asset prices. Holding monetary policy constant (green lines), the MPR shock raises 5-year nominal and real yields by 8 and 19 bp, respectively, on impact (so inflation compensation

[25] Again, near the steady state, these monetary-policy shocks have negligible effects on volatilities or risk premia, so the IRFs shown in the first two rows of the figure are essentially the same.

falls by 11 bp), and it reduces stock returns by about 1.5%. For comparison, these asset-price responses are quantitatively similar to those that would be induced by a monetary-policy shock of about 25 - 35 bp (see Figure 2). In addition, the shock causes the annualized conditional volatility of the stock market (our counterpart to the VIX index) to rise by about 100 bp and that of the 5-year yield (our counterpart to the MOVE) to rise by about 90 bp.[26]

Finally, Figure 4 shows how the term structures of volatility and interest rates react in the period when the MPR shock occurs. In the top panel, we show the response of the standard deviation of the one-period interest rate h periods ahead ($\text{std}_t[i_{t+h}]$). It decreases monotonically, with no substantial impact beyond a horizon of about three years. Even so, risk premia in the bond market respond non-monotonically across maturities. As shown in the lower panel, the response of the real term premium, which is generally the component of the yield curve that moves the most in response to MPR shocks, peaks at about 17 basis points at the two-year maturity, but it is elevated out to at least ten years. This response of real term premia contributes to the substantial steepening of the yield curve, as shown by the black line.[27]

5.4 Effects of MPR at the effective lower bound

The effective lower bound on nominal interest rates can have significant consequences for the asset-pricing effects of MPR. Because policy rates are essentially frozen in the near term, interest-rate volatility is lower at the ELB, and, all else equal, this has a dampening effect on risk premia. For the same reason, MPR shocks pass through less

[26]Our model also generates analogues to option-implied volatilities, but variance risk premia are small and respond only by negligible amounts to shocks. Therefore, for simplicity, we report the asset-price volatilities under the physical measure.

[27]In the figure, we show the response of the yield curve and its components without offsetting the effect of the MPR shock on the path of interest rates. Although that modification affects the expectations components of real rates and inflation, it has little effect on term premia at the steady state.

to risk premia across assets. In addition, because short-term rates cannot fall to offset the economic deterioration that occurs in response to greater uncertainty, output and inflation generally fall by more at the ELB than they do at the steady state. (This mirrors the standard result for aggregate-demand shocks in New Keynesian models.) The net effects of changes in risk premia, risk-free rates, and cash flows are qualitatively ambiguous.

Figure 5 shows the key impulse-responses when initiated at the ELB. Specifically, for the purposes of these plots, we initialize the IRF with values of i_{t-1} that would result in a short-term rate that would remain at the ELB for four quarters in the absence of shocks. All of the asset-price responses have the same sign at the steady state and at the ELB, but the magnitudes differ somewhat. Most notably, the impact of the MPR shock on contemporaneous stock returns rises from about 0.6% at the steady state to 0.8% at the ELB, again reflecting the larger multipliers associated with the absence of a policy response.

As shown in Figure 6, the effects of MPR shocks on the long end of the nominal yield curve are similar at the ELB to the steady-state case, but the short end of the curve cannot decline in this environment. The constrained nominal short rate, together with the amplified effect of MPR on inflation at the ELB, imply that the path of real rates rises significantly after the shock, in contrast to the steady-state case.

The responses of asset prices to MPR shocks at the ELB provide a validation test of the model. In Table 2, we showed event-study results for MPR shocks in the data during a sample when policy rates were constrained at zero. Table 9 compares those coefficients to the analogous responses in the model. (Recall that the model was calibrated only to the data responses at the steady state and relied only on the non-ELB subsample of the data.) As noted earlier, in the data, the coefficients all rise substantially at the ELB, with the responses of most bond yields to MPR shocks

increasing by a factor of roughly two and the response of equity prices increasing by a factor of more than 5. In the model, responses differ depending on just how far below zero the shadow rate initially is. To give a sense of the range of possible values, we show model responses for a values for $\hat{i}_0 = -1\%$ and $\hat{i}_0 = -4\%$ in the table.

For shadow rates in this range, the model-implied responses of stock returns and bond yields to MPR shocks are qualitatively—and, for the most part, quantiativey—consistent with the empirical findings that the responses of asset prices to MPR shocks increase substantially at the ELB. For example, when the shadow rate is at -4%, an MPR shock that raises one-year interest-rate volatility by 100 bp results in an increase in the five-year nominal yield of 153 bp and a decline in the stock market of 23.6%. These responses are quite close to the values of 162 bp and 25.9%, respectively, estimated in the event studies, and they are two to three times as large as the responses we observed near the steady state (see Table 7). Thus, the model accounts for the large increase in the relative importance of MPR when short-term policy rates are constrained at zero.

6 Case Study: Monetary Policy Risk around the 2022 Liftoff

In this section, we present a recent case study of how changes to MPR can be important for asset prices. As we showed in Figure 1 in the Introduction, surveys of market participants imply that interest-rate uncertainty increased dramatically between year-end 2021 and year-end 2023, strongly suggesting an increase in MPR. To gauge the effects of this episode on asset prices and the economy, we simulate a series of shocks that match this change in our model.

We begin by setting initial conditions to match the level of the federal funds rate

and the one-year-ahead uncertainty about its value (based on Figure 1), as of Q4 2021. At this point, the fed funds rate was still at the ELB, so the shadow rate could was negative. We assume that it was equal to -1%, implying an expected policy liftoff in Q1 2022, consistent with market expectations at the time. For simplicity, we assume that the macroeconomic variables and the value of the monetary-policy shock $\epsilon_{v,t}$ were initially at their steady-state values. We then simulate a series of eight quarterly shocks to η_t to generate the observed paths of the one-year-ahead uncertainty in the federal funds rate over the subsequent two years. Recall from Figure 1 that this uncertainty rose by 74 bp during the two-year period. (Since we only have the survey data on interest-rate uncertainty at year ends, we interpolate to obtain quarterly series.) We compare the outcomes in this simulation to a counterfactual economy in which there were no η_t shocks and MPR simply drifted back to its steady state from its Q4 2021 value.

Figure 7 shows the results. Bond yields rise monotonically during the period of rising MPR. By the end of the eight-quarter simulation, the nominal ten-year yield is about 30 bp higher, and the real ten-year yield is about 75 bp higher. Meanwhile, the stock-market falls by a cumulative 12%, and implied volatilities of both equities and interest rates increase.

We note that this exercise is likely to *understate* the size and effects of the MPR shock during this period. This is because, in the model, uncertainty about output and inflation rise in response to the MPR shock, as shown in the second row of the table. In order for macroeconomic uncertainty to remain constant, as Figure 1 shows it did in reality, some other structural source of risk must have declined at the same time the MPR was rising. In particular, as the disruptions resulting from the Covid-19 pandemic receded and supply chains were repaired, a number of contemporary reports suggested a decline in supply-side uncertainty. Since these forces were also endogenously reducing

uncertainty about interest rates, it must be the case that the increase in MPR was even larger than Figure 1 suggests. Thus, our results may be interpreted as a lower bound on the effects of the MPR shocks during this period.

It is possible to formulate alternative explanations for the rise in interest-rate uncertainty during this period. In particular, Bauer et al. (2024) argue that the market's perception of the inflation coefficient in the policy rule (ϕ_π in our notation) rose after the pandemic. If so, that would imply a higher level of interest rate uncertainty for any given amount of uncertainty in inflation, all else equal. However, the changes documented by Bauer et al. (2024) are not quantitatively large enough to explain the increase in uncertainty shown in Figure 1. Based on their estimates, we can perform the following calculation. As of the end of 2021, both policy-rule coefficients were approximately zero (see their Figure 2). Thus, MPR must have accounted for the entirety of interest-rate uncertainty, which our survey evidence shows was 33 bp at the one-year horizon. At the end of 2023, Bauer et al. estimate approximately $phi_\pi = 1$ and $\phi_y = 0.3$, using annualized rates. Given the survey measures of inflation uncertainty at 0.53% and output uncertainty at 1.1% at this time, and approximating the covariance between inflation and output as zero, we have

$$Var[i] \approx 1.07^2 = 1^2 0.53^2 + 0.3^2 1.1^2 + MPR^2 \tag{39}$$

This produces a contribution of MPR to the standard deviation of the interest rate one year ahead of 0.87%. In other words, even after accounting for a plausible increase in the perceived policy rule coefficients, we still attribute 54 basis points of the increase in one-year interest-rate uncertainty to higher MPR. Again, this value would be even larger if we were able to properly account for declining uncertainty about other shocks in the economy during this time.[28]

[28]We use headline PCE inflation, which is all that is reported in the primary dealer survey at

We conclude that an increase in uncertainty about the monetary-policy reaction function contributed to the substantial decline in asset prices during the post-Covid policy tightening. For comparison, in the data, 10-year nominal bond yields rose about 240 basis points over this period, inplying a 24% decline in zero-coupon prices. Meanwhile, 10-year TIPS yields rose about 270 basis points. Although some of that rise likely had to do with the tightening of monetary policy, measures of term premia, including those of Kim and Orphanides (2007), Adrian, Crump, and Moench (2020), and D'Amico, Kim, and Wei (2018) also rose significantly. The S&P 500 index of equity prices trailed its average two-year return by about 25 percentage points. Of course, many other shocks affected the economy during this period, so we cannot be more precise in comparing the simulation to the data. But, the analysis shows that the increase in MPR is directionally consistent with, and quantitatively accounts for a significant portion of, the net changes in asset prices that we observed in 2022 and 2023.

7 Conclusion

We have shown how risk surrounding the exogenous component of monetary policy contributes to risk premia across asset classes. Empirically, this kind of risk contributes significantly to the observed return volatility across asset classes and generates sizable price movements in event studies. The qualitative effects of MPR on risk premia, asset prices, and macroeconomic aggregates can be rationalized in a broad class of models where money is not neutral. Quantitatively, in a benchmark New Keynesian model with Epstein-Zin preferences, MPR explains significant portions of the empirical values

the one-year horizon. The calculation conservatively assumes that all uncertainty about GDP one year ahead is due to the output gap, not to potential output. If inflation and GDP were viewed as negatively correlated, as seems plausible during this period with prevalent supply-side dynamics, that would increase the calculated contribution of MPR even further.

of the equity premium and nominal term premium, more than accounts for the real term premium, and contributes negatively to the inflation-risk premium. At business-cycle frequencies, an increase in monetary-policy risk leads to declines in output and inflation; a rise in the equity premium; a fall in stock prices; an increase in real and nominal term premia increase; a steeper nominal yield curve; positive excess returns on nominal bonds and negative abnormal returns on real bonds; and a lower inflation-risk premium. Most of these effects are quantitatively significant. The ELB increases the magnitudes of these responses, also in line with the data.

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Tables and Figures

Table 1: Comparison of Asset-Price Volatility on Blackout and Non-Blackout Days

A. Full sample

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	5.8	6.3	8.3	6.2	18.6
10Y nom. yield	5.5	6.0	7.4	5.9	17.0
5Y real yield	5.4	6.2	9.8	5.9	21.5
10Y real yield	4.8	5.4	8.5	5.2	18.3
SPX	109	115	127	115	304
VIX	154	160	203	157	403
MOVE	3.8	4.3	4.5	4.2	13.2
Sample Size	1155	5267	258	5009	

B. Short rate above ELB

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	6.1	6.6	8.0	6.5	18.3
10Y nom. yield	5.6	6.0	6.5	6.0	17.2
5Y real yield	5.7	6.3	8.9	6.0	19.6
10Y real yield	4.8	5.3	7.3	5.1	17.1
SPX	107	114	124	113	336
VIX	143	150	168	148	445
MOVE	3.7	4.1	4.3	4.1	12.3
Sample Size	796	3741	185	3556	

C. Short rate at ELB

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	4.9	5.6	9.2	5.3	18.2
10Y nom. yield	5.5	5.9	9.3	5.7	16.2
5Y real yield	5.0	6.1	11.4	5.6	25.0
10Y real yield	4.9	5.7	10.6	5.3	20.1
SPX	115	119	134	118	218
VIX	176	183	273	177	358
MOVE	4.1	4.6	4.7	4.6	15.4
Sample Size	359	1526	73	1543	

Note: Daily data, 1995-2024, except for real yields, which begin in 2003. All values are reported in basis points. The final column reports the counterfactual standard deviation of each variable when all sources of risk other than monetary policy are shut down, using equation (7), rescaled to a quarterly frequency.

Table 2: Responses of Asset Prices to Short-Rate Uncertainty around FOMC Announcements

A. Pooled sample

	5Y nom. yield	10Y nom. yield	5Y real yield	10Y real yield	SPX	VIX	MOVE
Monetary-policy shock	0.40***	0.19***	0.63***	0.38***	-2.80**	1.10	-0.04
	(0.07)	(0.07)	(0.11)	(0.10)	(1.25)	(1.80)	(0.04)
Short-rate uncertainty (MPR)	0.87***	0.87***	1.55***	1.30***	-8.54***	20.6***	0.69***
	(0.16)	(0.15)	(0.20)	(0.10)	(2.78)	(4.10)	(0.09)
Adj. R^2	0.29	0.20	0.40	0.31	0.07	0.11	0.20
# obs.	243	243	170	170	244	244	240

B. Allowing for break at ELB

		5Y nom. yield	10Y nom. yield	5Y real yield	10Y real yield	SPX	VIX	MOVE
Monetary -policy shock	Above ELB	0.39***	0.19***	0.54***	0.35***	-2.99**	1.7	-0.04
		(0.07)	(0.07)	(0.12)	(0.1)	(1.31)	(1.9)	(0.04)
	Marginal Effect of ELB	-0.09	-0.09	0.15	-0.09	4.33	-9.3*	-0.01
		(0.21)	(0.20)	(0.27)	(0.24)	(3.71)	(5.4)	(0.13)
Short-rate uncertainty (MPR)	Above ELB	0.69***	0.65***	1.21***	0.98***	-4.69	14.5***	0.67***
		(0.17)	(0.16)	(0.25)	(0.23)	(2.93)	(4.3)	(0.10)
	Marginal Effect of ELB	0.93***	1.08***	0.94**	1.08***	-21.20***	35.8***	0.11
		(0.35)	(0.33)	(0.45)	(0.41)	(6.14)	(9.0)	(0.21)
Adj. R^2		0.31	0.24	0.42	0.33	0.12	0.16	0.19
# obs.		243	243	170	170	244	244	240

Note: Notes: Daily data, 1995 – 2024, except for real yields, which begin in 2003. All values are reported in basis points. Standard errors in parentheses. Asterisks indicate statistical significance at the 10% (*), 5% (**), and 1% (***) level.

Table 3: Assumptions about Qualitative Responses to Shocks in the Log-Linear Model

	Monetary-policy shocks	Supply shocks	Demand shocks
Output	-	+	+
Inflation	-	-	+
Nominal short rate	+	??	+
Stochastic discount factor	+	-	-

Table 4: Effects of Structural Risks on Risk Premia

	Monetary-policy shocks	Supply shocks	Demand shocks
Nominal term premium	+	+	-
Real term premium	+	+	-
Inflation risk premium	-	+	-
Equity premium	+	+	+

Note: Response of term premia to supply-side risk assumes that parameter values are such that positive supply shocks cause nominal short-term interest rates to fall.

Table 5: Parameter Values in Baseline Model

Parameter	Interpretation	Value	Source
β	Time preference	0.99	Standard
ρ	Inverse IES	0.5	Standard
ψ	Labor share in production	0.35	Standard
κ	Philips Curve slope	0.05/4	Standard
ϕ_π	Policy response to inflation	1.5	Standard
ϕ_y	Policy response to output gap	0.5/4	Standard
r^*	Steady-state real rate	0	Average of data
π^*	Steady-state inflation	0.019/4	Average of data
α	Risk aversion	-118	Match empirical results
ϕ_i	Policy rule inertia	0.67	Match empirical results
ρ_ν	Persistence of monetary policy shocks	0.75	Match empirical results
ζ	Steady-state MPR	0.0088/4	Match empirical results
ρ_ζ	Persistence of MPR shocks	0.75	Match empirical results
σ_ν	Std. dev. MPR shocks	0.0030/4	Match empirical results
δ	Equity leverage	0.79	Match empirical results

Note: Time interval is taken to be quarterly.

Table 6: Model match to blackout-period evidence: MPR contribution to quarterly volatility

	Stock return	5y nom. yield (bps)	5y real yield (bps)	10y nom. yield (bps)	10y real yield (bps)	VIX (ann. %)	MOVE (ann. bps)
Data	3.4	18	20	17	17	4.5	12
Model	1.9	26	39	13	20	1.5	16

Note: Data values are calculated from daily market volatilities during blackout periods, relative to non-blackout dates, scaled to a quarterly frequency, excluding observations at the ELB. (See Table 1.) For the purposes of comparison, the MOVE index is matched to the volatility of the model-based five-year nominal yields, and variance risk premia are assumed constant.

Table 7: Model match to event-study evidence

A. Response to monetary-policy shock on impact, holding 1-year rate uncertainty constant

	5y nom. yield (bps)	5y real yield (bps)	10y nom. yield (bps)	10y real yield (bps)	Stock return (%)	VIX (ann. %)	MOVE (ann. bps)
Data	+39	+54	+19	+35	-3.0	+1.7	-4
	(7)	(12)	(7)	(11)	(1.3)	(1.9)	(4)
Model	+34	+52	+17	+26	-3.9	-0.4	-3

B. Response to MPR shock on impact, holding 1-year short rate constant

	5y nom. yield (bps)	5y real yield (bps)	10y nom. yield (bps)	10y real yield (bps)	Stock return (%)	VIX (ann. %)	MOVE (ann. bps)
Data	+69	+121	+65	+98	-4.7	+14.5	+67
	(17)	(25)	(16)	(23)	(2.9)	(4.3)	(10)
Model	+50	+131	+26	+67	-10.3	+9.3	+81

Note: Data values are calculated from the coefficients estimated in an event study around FOMC days, following Bauer et al. (2022), excluding observations at the ELB (see Table 2). Standard errors are reported in parentheses. In Panel A, model values are calculated as the effects of a 1 bp shock to monetary policy, divided by the effect of this shock on the one-year forward interest rate, neutralizing the effect on the one-year rate uncertainty with an offsetting shock to MPR. In Panel B, model values are calculated as the effects of a 1 bp shock to MPR, divided by the effect of this shock on 1-year rate uncertainty, neutralizing the effect on the one-year yield with an offsetting shock to monetary policy. For the purposes of comparison, the MOVE index is matched to the volatility of the model-based five-year nominal yields, and variance risk premia are assumed constant.

Table 8: Model contributions to selected risk premia

	10y nom. term prem. (bps)	10y real term prem. (bps)	10y inflation risk prem. (bps)	Equity premium (bps, annualized)
MPR (ζ)	62	95	-32	40
MPR volatility (σ_ζ)	-5	2	-8	3
Total	57	97	-40	43

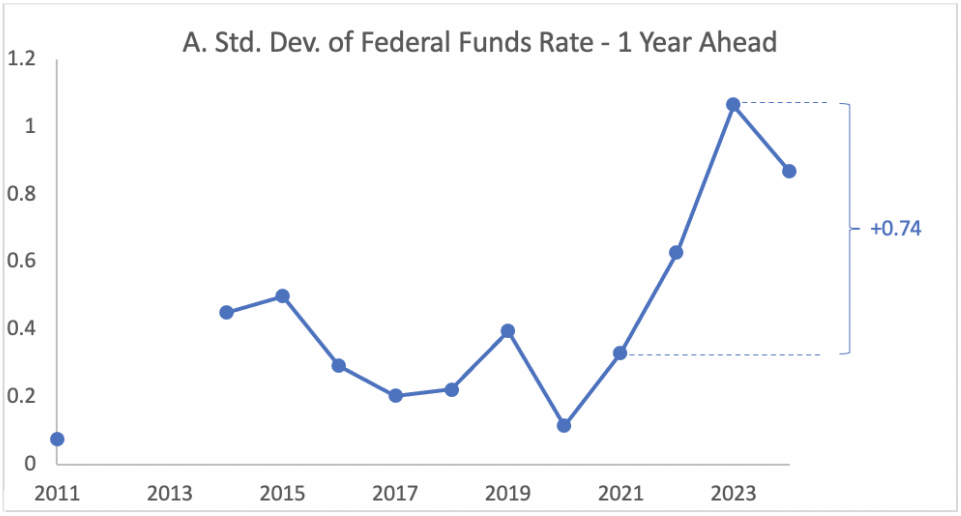
Table 9: Responses to shocks: data vs. model, at the ELB

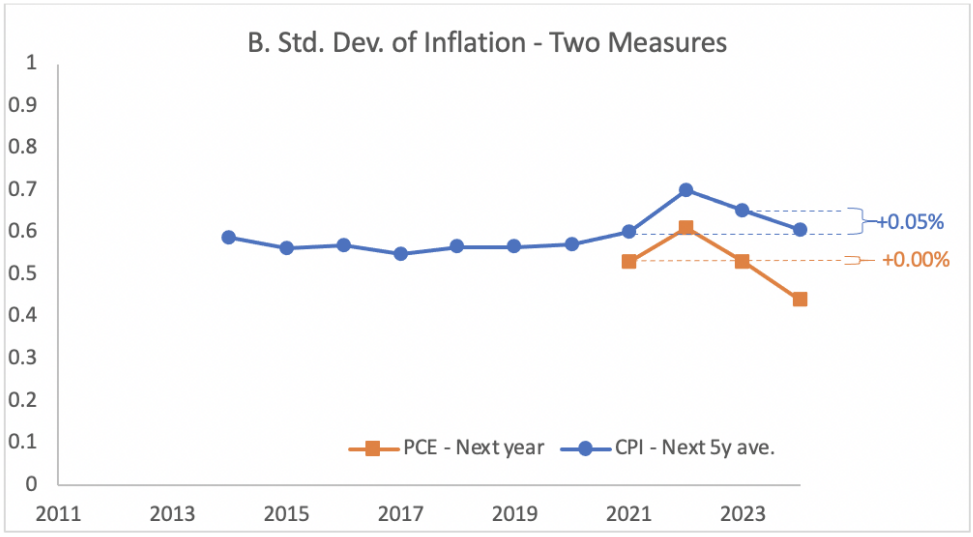
A. Response to MPR shock on impact, holding 1-year short rate constant

		5y nom. yield (bps)	5y real yield (bps)	10y nom. yield (bps)	10y real yield (bps)	Stock return (%)	VIX (ann. %)	MOVE (ann. bps)
Data, $i_t = 0$		1.62	2.15	1.73	2.06	-25.9	50.3	0.78
Model	$\hat{i}_{(t-1)} = -1\%$	2.76	4.67	1.39	2.35	-37.50	2.32	0.40
	$\hat{i}_{(t-1)} = -4\%$	1.53	2.85	0.77	1.43	-23.63	0.84	0.13

Note: Data values are calculated from the coefficients estimated in an event study around FOMC days, following Bauer et al. (2022), restricted to observations at the ELB. Model values simulate a shock to MPR that moves the one-year rate uncertainty by 1%, neutralizing the effect on the one-year yield with an offsetting shock to monetary policy, starting from shadow rate values of 0 or -4%. For the purposes of comparison, the MOVE index is matched to the volatility of the model-based five-year nominal yields, and variance risk premia are assumed constant.

Figure 1: Uncertainty Measures from December Survey of Primary Dealers





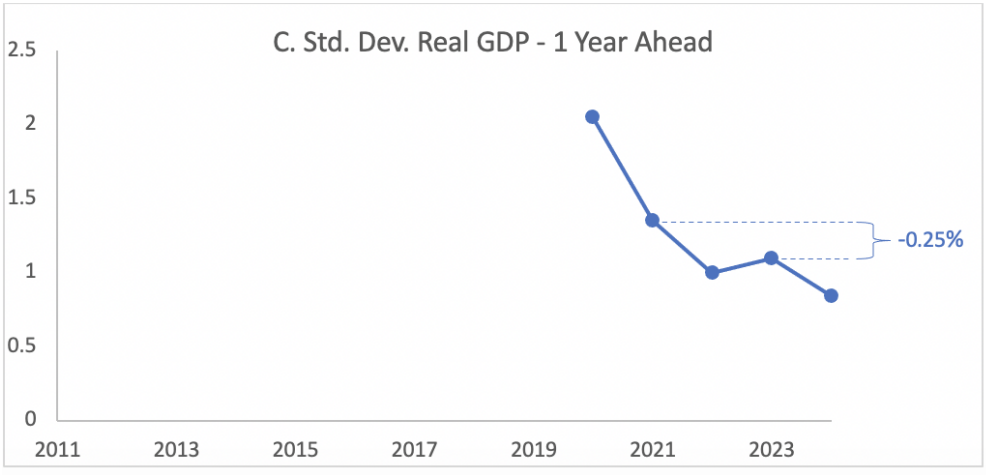


Figure 2

Response to a MP shock calibrated to produce a 25 b.p. short rate response, without ZLB, from steady state, % annualized

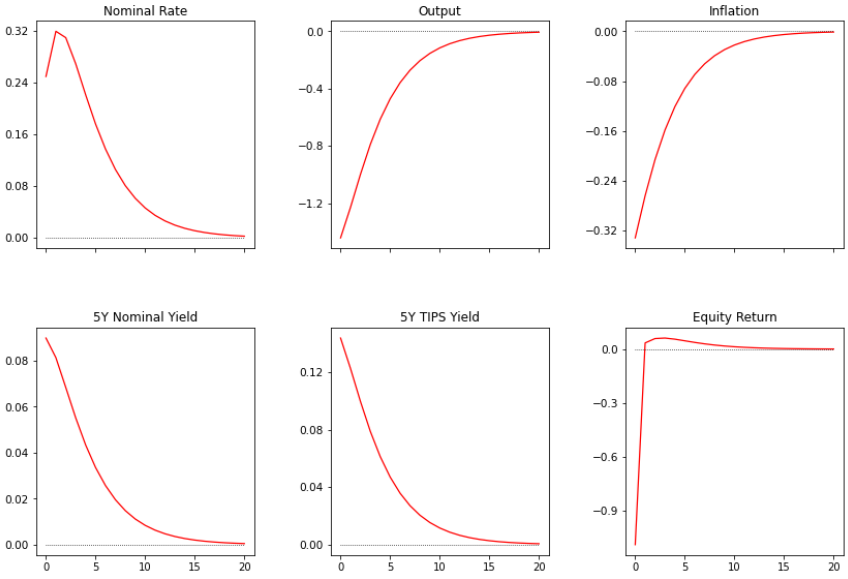


Figure 3

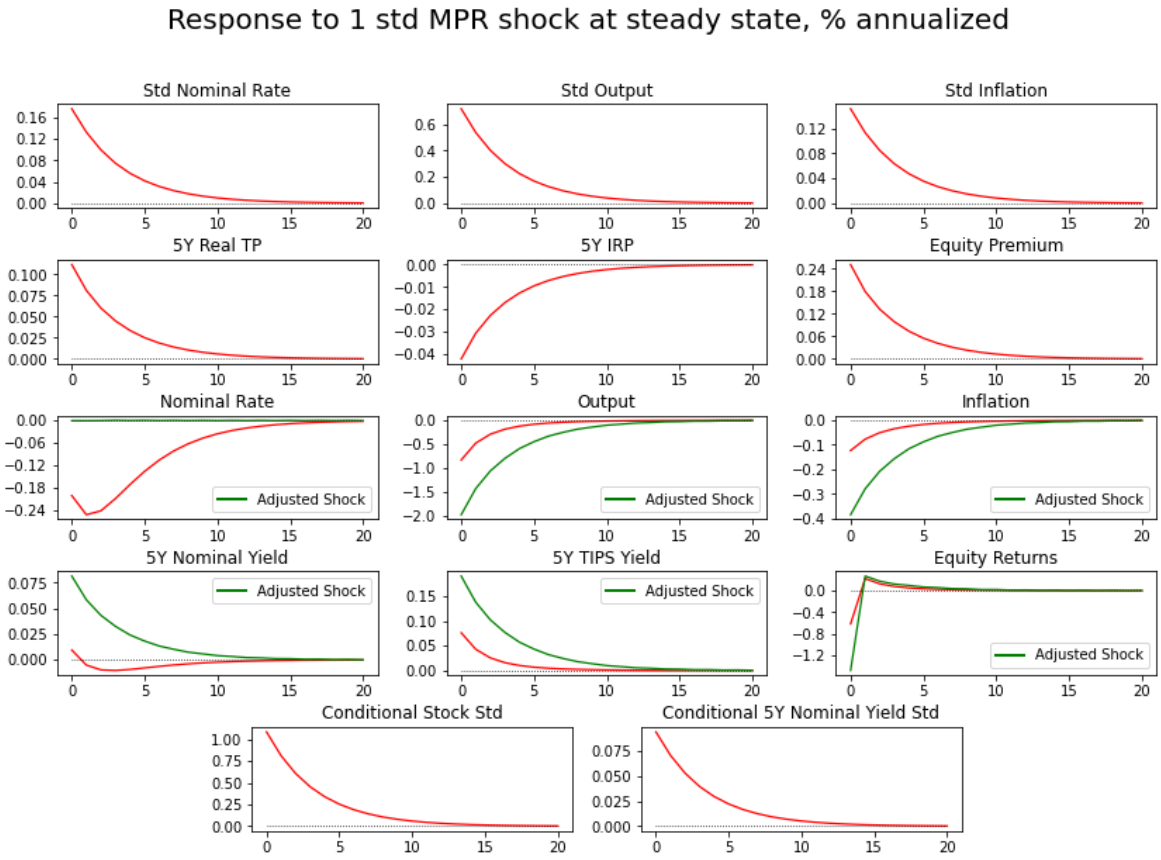

Response to 1 std MPR shock at steady state, % annualized
Std Nominal Rate
Std Output
Std Inflation
5Y Real TP
5Y IRP
Equity Premium
Nominal Rate
Output
Inflation
5Y Nominal Yield
5Y TIPS Yield
Equity Returns
Conditional Stock Std
Conditional 5Y Nominal Yield Std
Adjusted Shock

Figure 4: Response of interest-rate uncertainty and the yield curve in the period of an MPR shock

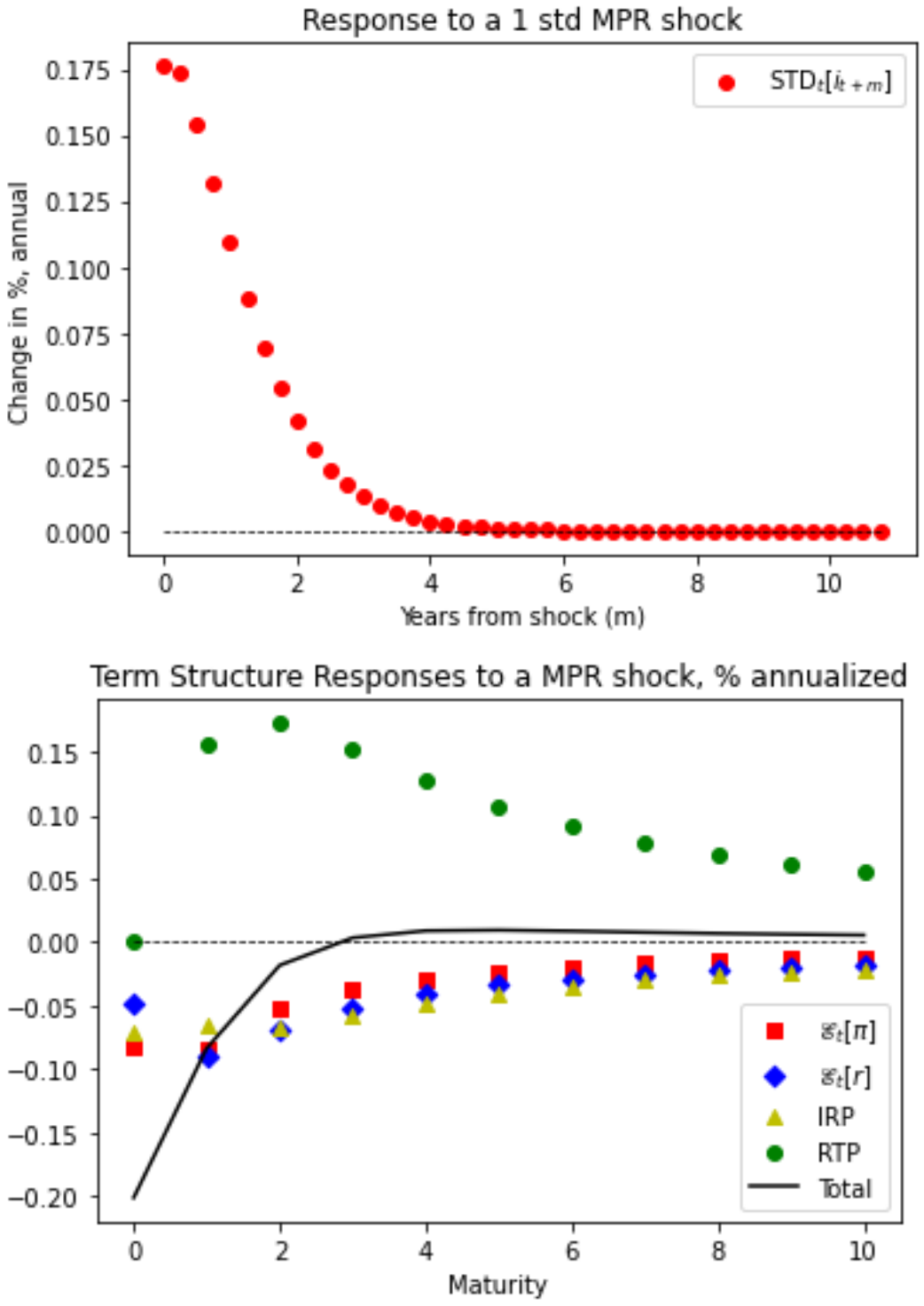


Figure 5

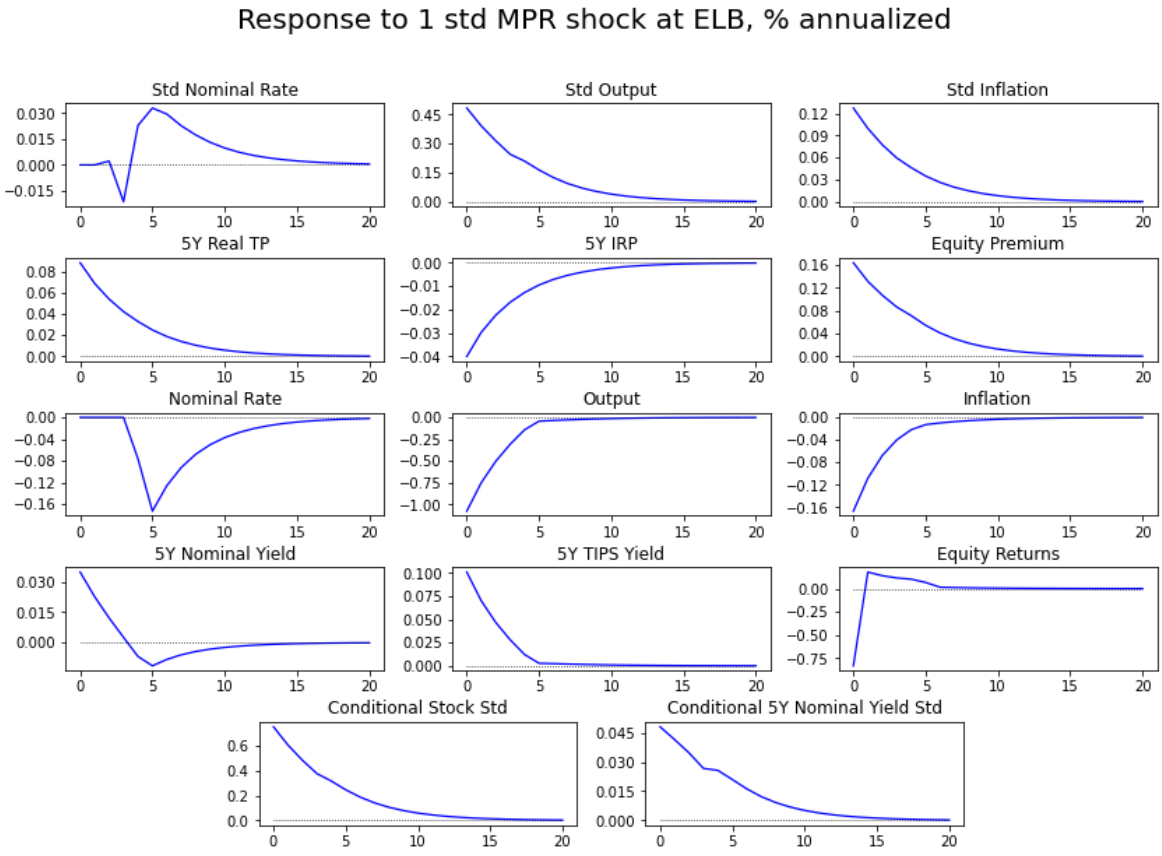

Response to 1 std MPR shock at ELB, % annualized
Std Nominal Rate
Std Output
Std Inflation
5Y Real TP
5Y IRP
Equity Premium
Nominal Rate
Output
Inflation
5Y Nominal Yield
5Y TIPS Yield
Equity Returns
Conditional Stock Std
Conditional 5Y Nominal Yield Std

Figure 6

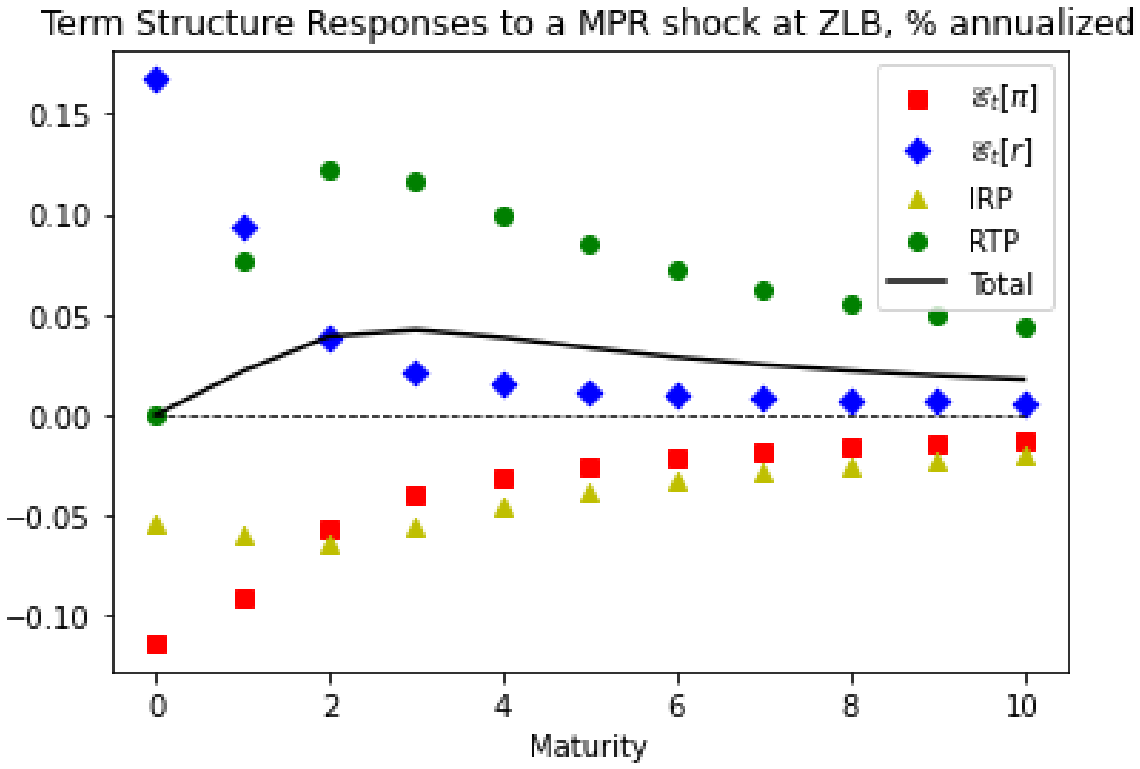
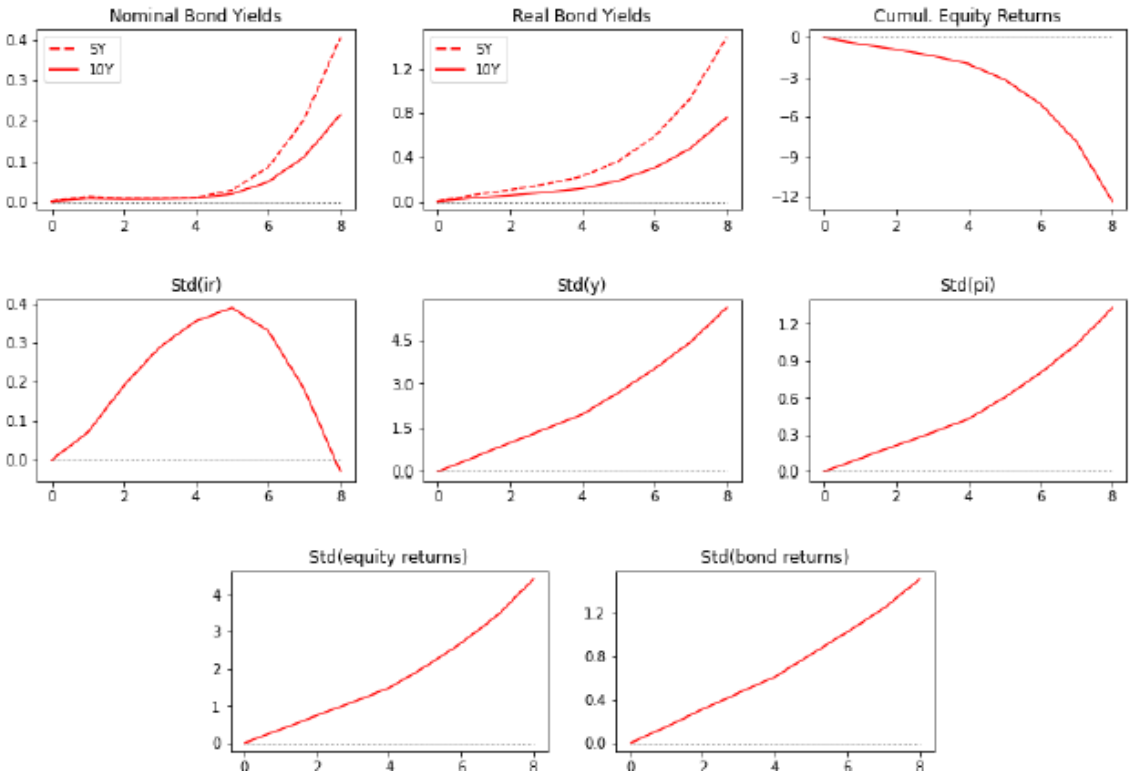

Term Structure Responses to a MPR shock at ZLB, % annualized
0.15
0.10
0.05
0.00
−0.05
−0.10
0
2
4
6
8
10
Maturity
$\mathbb{E}_t[\pi]$
$\mathbb{E}_t[r]$
IRP
RTP
Total

Figure 7: Simulations of selected variables in response to MPR shock matching the 2022-23 data



APPENDICES

A Proofs of Propositions

A.1 Proof of proposition 1

Taking a first-order approximation of asset returns,

$$\begin{aligned} R_t &= f(v_t, \epsilon_{x,t}, \zeta_t) - f(v_{t-1}, \epsilon_{x,t-1}, \zeta_{t-1}) && (40) \\ &\approx \frac{\partial f}{\partial v_t}[(\rho_v - 1)v_{t-1} + \zeta_{t-1}\epsilon_{v,t}] + \frac{\partial f}{\partial x_t}\epsilon_{x,t} + \frac{\partial f}{\partial \zeta_t}[(\rho_\zeta - 1)(\zeta_{t-1} - \bar{\zeta}) + \eta_t] && (41) \end{aligned}$$

Taking macroeconomic shocks to be zero and assuming $\rho_v \approx \rho_\zeta \approx 1$, this simplifies to

$$R_t|_{\epsilon_{x,t}=0} \approx \frac{\partial f}{\partial v_t}(\zeta_{t-1}\epsilon_{v,t}) + \frac{\partial f}{\partial \zeta_t}\eta_t \tag{42}$$

so

$$\text{var}\left[R_t|\epsilon_{x,t} = 0\right] \approx \left(\frac{\partial f}{\partial v_t}\right)^2 \text{var}[\zeta] + \left(\frac{\partial f}{\partial \zeta_t}\right)^2 \sigma_t^2 \tag{43}$$

Meanwhile, on blackout days,

$$\text{var}\left[R_t|\epsilon_{v,t} = \eta_t = 0\right] \approx \left(\frac{\partial f}{\partial x_t}\right)^2 \sigma_x^2 \tag{44}$$

Since the three shocks are independent, equation (7) follows.

A.2 Asset Pricing Derivations

We define the operator $\mathscr{E}_t^{\circ(N)}$ as the time-t log expectation of the N-period geometric mean. That is, for any random variable x_t,

$$\mathscr{E}_t^{\circ(N)}[x_s] \equiv \log E_t\left[\exp\left(\frac{\sum_{n=1}^N x_{s+n-1}}{N}\right)\right]. \tag{45}$$

We work with geometric expectations, rather than the arithmetic expectations because they subsume Jensen's inequality terms, which are not of primary interest, into physical expectations of future prices and cash flows and leave risk premia as clean measures of deviations from risk neutrality. Under the assumption of log normality that we maintain in this section,

$$\mathscr{E}_t^{(N)}[x_s] = E_t\left[\frac{1}{N}\sum_{n=1}^{N} x_{s+n-1}\right] + \frac{1}{2}\text{var}_t\left[\frac{1}{N}\sum_{n=1}^{N} x_{s+n-1}\right]. \tag{46}$$

From equation (10), the short-term real interest rate is equal to the negative log expectation of the one-period SDF, as is standard:

$$r_t = -E_t[m_{t+1}] - \frac{\sigma_m^2}{2} = -\mathscr{E}_t^{(1)}[m_{t+1}] \tag{47}$$

Similarly, the nominal interest rate satisfies

$$\begin{aligned} i_t &= -\mathscr{E}_t^{(1)}[m_{t+1} - \pi_{t+1}] && (48)\\ &= r_t + \mathscr{E}_t^{(1)}[\pi_{t+1}] + \underbrace{\text{cov}_t[m_{t+1}, \pi_{t+1}]}_{IRP^{(1)}}. && (49) \end{aligned}$$

In equilibrium, the real interest rate adjusts such that both the policy rule (1) and the no-arbitrage condition (49) are always satisfied.

The two period real yield is given by:

$$\begin{aligned} y_t^{(2)} &= -\frac{1}{2}\log \text{E}_t\left[\exp\left(m_{t+1} + m_{t+2}\right)\right] && (50)\\ &= \mathscr{E}_t^{(2)}\left[r_t\right] + \underbrace{\text{cov}_t[m_{t+1}, r_{t+1}]}_{RTP_t^{(2)}}. && (51) \end{aligned}$$

The two-period nominal yield can be written as

$$\begin{aligned} y_t^{\$(2)} &= \mathscr{E}_t^{(2)}[i_t] + \text{cov}_t[m_{t+1} - \pi_{t+1}, i_{t+1}] && (52)\\ &= \mathscr{E}_t^{(2)}[r_t] + \mathscr{E}_t^{(2)}[\pi_t] + RTP_t^{(2)} \\ &\quad + \underbrace{IRP_t^{(1)} + \text{cov}_t[\pi_{t+1}, m_{t+2}] + \text{cov}_t[\pi_{t+2}, m_{t+1}] + \text{cov}_t[\pi_{t+2}, m_{t+2}]}_{IRP_t^{(2)}} && (53) \end{aligned}$$

The stock price is

$$P_t^{eq} = \mathrm{E}_t\left[\exp(m_{t+1} + y_{t+1})\right] \tag{54}$$

The log nominal return on this asset is given by

$$r_{t+1}^{eq} + \pi_{t+1} \;=\; y_{t+1} - \mathscr{E}_t^{(1)}\left[m_{t+1} - \pi_{t+1} + y_{t+1}\right] \tag{55}$$

A.3 Proof of proposition 2

The variance of nominal interest rates is given by

$$\begin{aligned}\mathrm{var}[i_{t+1}] &= \boldsymbol{\phi}'\mathrm{var}[\mathbf{x}_{t+1}]\boldsymbol{\phi} + \zeta_t\boldsymbol{\phi}'\mathrm{cov}_t[\mathbf{x}_{t+1}, \epsilon_{v,t+1}] + \zeta^2 \\ &= \boldsymbol{\phi}'\left(\zeta^2\mathbf{k}'\mathbf{k} \;+ \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + \zeta^2\left(2\boldsymbol{\phi}'\mathbf{k} + 1\right)\end{aligned}$$

making use of the normalizations that all shocks are uncorrelated and have unit variance. Because we have assumed $\boldsymbol{\phi}'\mathbf{k} > -1$ (ensuring that the policy rate moves in the same direction as the monetary shock), the variance of nominal short rates unambiguously rises with MPR. Meanwhile, from equation (49), the real short rate is

$$\begin{aligned} r_t &= i_t - \mathscr{E}_t^{(1)}[\pi_{t+1}] - \mathrm{cov}_t[m_{t+1}, \pi_{t+1}] &(56)\\ &= i_t - \mathrm{E}_t[\pi_{t+1}] + \frac{1}{2}\mathrm{var}[\pi_{t+1}] - \mathrm{cov}_t[m_{t+1}, \pi_{t+1}] &(57)\end{aligned}$$

So $\mathrm{var}_t[r_{t+1}] = \mathrm{var}_t[i_{t+1}] + \mathrm{var}_t[\mathrm{E}_{t+1}[\pi_{t+2}]] - \mathrm{cov}_t[i_{t+1}, \mathrm{E}_{t+1}[\pi_{t+2}]]$. This gives

$$\mathrm{var}_t[r_{t+1}] = \boldsymbol{\phi}'\left(\zeta^2\mathbf{k}'\mathbf{k} \;+ \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + \zeta^2\left(2\boldsymbol{\phi}'\mathbf{k} + 1\right) + k_\pi^2\rho^2\zeta^2 - k_\pi\zeta^2 \tag{58}$$

Since k_π is negative by assumption (positive monetary-policy shocks reduce inflation), higher levels of MPR are associated with a higher variance of short-term real interest rates.

Similarly, with constant second moments the variance of nominal bond yields is

$$\begin{aligned}\mathrm{var}_t[y_{t+1}^{\$(2)}] &= \mathrm{var}_t[i_{t+1}] + \mathrm{var}_t[E_{t+1}[i_{t+2}]] + 2\mathrm{cov}_t[i_{t+1}, i_{t+2}] &(59)\\ &= \boldsymbol{\phi}'\left(\zeta^2((1+\rho)^2+1)\mathbf{k}'\mathbf{k} \;+ \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + \zeta^2((1+\rho)^2+1)\left(\boldsymbol{\phi}'\mathbf{k} + 1\right) + 2\zeta^2\rho &(60)\end{aligned}$$

and the variance of real yields is

$$\begin{aligned}
\text{var}_t[y^{(2)}_{t+1}] &= \text{var}_t[r_{t+1}] + \text{var}_t[r_{t+2}] + 2\text{cov}_t[r_{t+1}, r_{t+2}] && (61)\\
&= \text{var}_t[i_{t+1}] + \text{var}_t[E_{t+1}[i_{t+2}]] + \text{var}_t[E_{t+1}[\pi_{t+2}]] + \text{var}_t[E_{t+1}[\pi_{t+3}]] && (62)\\
&\quad +2\left(\text{cov}_t[i_{t+1}, E_{t+1}[i_{t+2}]] - \text{cov}_t[i_{t+1}, E_{t+1}[\pi_{t+2}]] - \text{cov}_t[E_{t+1}[i_{t+2}], E_{t+1}[\pi_{t+3}]]\right) && (63)\\
&\quad +2\left(-\text{cov}_t[i_{t+1}, E_{t+1}[\pi_{t+3}]] - \text{cov}_t[E_{t+1}[i_{t+2}], E_{t+1}[\pi_{t+2}]] + \text{cov}_t[E_{t+1}[\pi_{t+2}], E_{t+1}[\pi_{t+3}]]\right) && (64)\\
&= \boldsymbol{\phi}'\left(\zeta^2\mathbf{k}'\mathbf{k} + \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + \zeta^2\left(2\boldsymbol{\phi}'\mathbf{k} + 1\right) + \rho^2\zeta^2 && (65)\\
&\quad +k_\pi^2\rho^2\zeta^2 + k_\pi^2\rho^4\zeta^2 + 2\left(\rho\zeta^2 - k_\pi\rho\zeta^2 - k_\pi\rho^3\zeta^2 - 2k_\pi\rho^2\zeta^2 + k_\pi^2\rho^3\zeta^2\right) && (66)
\end{aligned}$$

Again, recalling that $k_\pi < 0$ and $\boldsymbol{\phi}' k_\pi > -1$, both variances are strictly increasing in ζ.

Turning to the variances of asset returns, for an N-period nominal bond, the log price is, by definition, $p_t^{\$(N)} = -Ny_t^{\$(N)}$, and the one-period nominal return is $p_{t+1}^{\$(N-1)} - p_t^{\$(N)}$. Thus, the variance of the return is

$$\text{var}_t\left[-i_{t+1} + 2y_t^{\$(2)}\right] = \boldsymbol{\phi}'\left(\zeta^2\mathbf{k}'\mathbf{k} + \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + \zeta^2 \tag{67}$$

which is increasing in ζ. For inflation-indexed bonds, the analogous calculation is

$$\begin{aligned}
\text{var}_t\left[-r_{t+1} + 2y_t^{(2)} + \pi_{t+1}\right] &= \boldsymbol{\phi}'\left(\zeta^2\mathbf{k}'\mathbf{k} + \mathbf{A}'\mathbf{A}\right)\boldsymbol{\phi} + 2\boldsymbol{\phi}'\mathbf{k}(1-\rho)k_\pi\zeta^2 && (68)\\
&\quad +k_\pi^2(\rho-1)^2\zeta^2 && (69)
\end{aligned}$$

Again, this is increasing in ζ under our parameter restrictions.

From equation (55), the conditional variance of the return on the two-period equity claim is simply

$$\text{var}_t[y_{t+1}] = k_y^2\zeta^2 + (a_y^d\sigma_d)^2 + (a_y^s\sigma_s)^2 \tag{70}$$

which is clearly increasing in ζ.

From equation (12), the effect of MPR on the real term premium is $(1+\boldsymbol{\phi}'\mathbf{k}-\kappa_\pi)\mu^v$, which is strictly positive. From equation (53), the two-period inflation risk premium is

$$IRP_t^{(2)} = IRP_t^{(1)} + \text{cov}_t[\pi_{t+1}, m_{t+2}] + \text{cov}_t[\pi_{t+2}, m_{t+1}] + \text{cov}_t[\pi_{t+2}, m_{t+2}]$$

and, from equation (11), we have $IRP^{(1)} = a_\pi^s\mu^s + a_\pi^d\mu^d + \kappa_\pi\mu^v\zeta_{t-1}^2$. Meanwhile it is

straightforward to show

$$\begin{aligned}
\text{cov}_t[\pi_{t+1}, m_{t+2}] &= 0 && (71)\\
\text{cov}_t[\pi_{t+2}, m_{t+1}] &= \kappa_\pi \rho \mu^v \zeta^2 && (72)\\
\text{cov}_t[\pi_{t+2}, m_{t+2}] &= \mu^d a^d + \mu^s a^s + \kappa_\pi \mu^v \zeta^2 && (73)
\end{aligned}$$

Consequently, the effect of an increase in ζ^2 on $IRP^{(2)}$ is $\kappa_\pi \mu^v (\rho + 2)$, which is strictly negative.

The two-period nominal term premium is

$$\begin{aligned}
NTP_t^{(2)} &= \text{cov}_t[m_{t+1} - \pi_{t+1}, i_{t+1}] && (74)\\
&= (\phi_y a_y^d + \phi_\pi a_\pi^d)(\mu^d - a_\pi^d) + (\phi_y a_y^s + \phi_\pi a_\pi^s)(\mu^s - a_\pi^d) && (75)\\
&\quad + (1 + \boldsymbol{\phi}' \mathbf{k})(\mu^v - k_\pi)\zeta_t^2 && (76)
\end{aligned}$$

Since $\mu^v > 0$ and $k\pi < 0$, the effect of ζ is strictly positive.

Finally, the equity premium in the two-period log-normal model is given in equation (15). Since $\mu^v > 0$ and $k_y < 0$, the effect of ζ in this equation is strictly positive.

B Robustness Checks of Empirical Results

This appendix reports the robustness of the empirical results to dropping observations during periods of extreme financial stress and unusual Fed activity. In particular, we drop all observations between September 2008 and March 2009, corresponding to the most acute phase of the global financial crisis and the Fed's intervention into funding, Treasury, and mortgage markets. We also drop March 2020, the month with the extraordinary financial turmoil (and Fed response) associated with the Covid-19 pandemic. Excluding these observations reduces our sample size by about 2%.

The qualitative patterns documented in our comparison of blackout days to non-blackout days (Table A1) and in our event study around FOMC announcements (Table A2) remain the same as in our baseline results. Quantitatively, most results are similar as well, although there is some decline in the estimated effects of MPR shocks on real yields.

Table 1. Comparison of Asset-Price Volatility on Blackout and Non-Blackout Days

A. Full sample

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	5.8	6.3	7.9	6.2	18.6
10Y nom. yield	5.5	6.0	7.1	5.9	17.0
5Y real yield	5.4	6.2	9.9	5.9	21.5
10Y real yield	4.8	5.5	8.3	5.2	18.3
SPX	109	117	119	116	304
VIX	154	164	152	155	403
MOVE	3.8	4.3	4.3	4.2	13.2

B. Short rate above ELB

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	6.1	6.6	7.3	6.5	18.3
10Y nom. yield	5.5	6.0	5.9	6.0	17.2
5Y real yield	6.0	6.3	8.7	6.0	19.6
10Y real yield	5.3	5.9	5.9	5.8	17.1
SPX	106	116	112	115	336
VIX	143	156	156	152	445
MOVE	3.7	4.1	4.2	4.1	12.3

C. Short rate at ELB

	Blackout	Non-blackout			Implied qtrly std. dev. due to MPR
		All non-blackout days	FOMC days	Non-FOMC days	
5Y nom. yield	4.9	5.6	9.2	5.3	18.2
10Y nom. yield	5.5	5.9	9.3	5.7	16.2
5Y real yield	5.0	6.1	11.4	5.6	25.0
10Y real yield	4.9	5.7	10.6	5.3	20.1
SPX	115	119	134	118	218
VIX	176	183	273	177	358
MOVE	4.1	4.6	4.7	4.6	15.4

Notes: Daily data, 1995 – 2024, except for real yields, which begin in 2003. All values are reported in basis points. The final column reports the counterfactual standard deviation of each variable when all sources of risk other than monetary policy are shut down, using equation (7), rescaled to a quarterly frequency.

Table 2. Responses of Asset Prices to Short-Rate Uncertainty around FOMC Announcements

A. Pooled sample

	5Y nom. yield	10Y nom. yield	5Y real yield	10Y real yield	SPX	VIX	MOVE
Short-rate uncertainty (MPR)	0.87*** (0.16)	0.87*** (0.15)	1.55*** (0.23)	1.30*** (0.20)	-8.54*** (2.78)	20.6*** (4.1)	0.69*** (0.09)
Monetary-policy shock	0.40*** (0.07)	0.19*** (0.07)	0.63*** (0.11)	0.38*** (0.10)	-2.80** (1.25)	1.1 (1.8)	-0.04 (0.04)
Adj. R^2	0.29	0.20	0.40	0.31	0.07	0.11	0.20
# obs.	243	243	170	170	244	244	240

B. Allowing for break at ELB

		5Y nom. yield	10Y nom. yield	5Y real yield	10Y real yield	SPX	VIX	MOVE
Short-rate uncertainty (MPR)	Above ELB	0.69*** (0.17)	0.65*** (0.16)	1.21*** (0.25)	0.98*** (0.23)	-4.69 (2.93)	14.5*** (4.3)	0.67*** (0.10)
	Marginal Effect of ELB	0.93*** (0.35)	1.08*** (0.33)	0.94** (0.45)	1.08*** (0.41)	-21.2*** (6.14)	35.8*** (9.0)	0.11 (0.21)
Monetary-policy shock	Above ELB	0.39*** (0.07)	0.19*** (0.07)	0.54*** (0.12)	0.35*** (0.11)	-2.99** (1.31)	1.7 (1.9)	-0.04 (0.04)
	Marginal Effect of ELB	-0.09 (0.21)	-0.09 (0.20)	0.15 (0.27)	-0.09 (0.24)	4.33 (3.71)	-9.3* (5.4)	-0.01 (0.13)
Adj. R^2		0.31	0.24	0.42	0.33	0.12	0.16	0.19
# obs.		243	243	170	170	244	244	240

Notes: Daily data, 1995 – 2024, except for real yields, which begin in 2003. All values are reported in basis points. Standard errors in parentheses. Asterisks indicate statistical significance at the 10% (*), 5% (**), and 1% (***) level.

C Equilibrium Conditions in the NK Model

C.1 Households

Denoting Lagrange multipliers by Λ_t, the first-order conditions for the household's problem of maximizing (18), subject to (19), are

$$\begin{aligned}
\frac{\partial U_t}{\partial C_t} &= (1-\beta)C_t^{\rho-1}U_t^{1-\rho} = -P_t\Lambda_t && (77)\\
\frac{\partial U_t}{\partial H_t} &= -(1-\beta)\theta_t\chi H_t^{\chi\rho-1}U_t^{1-\rho} = W_t\Lambda_t && (78)\\
Q_t &= E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\right] && (79)
\end{aligned}$$

These imply the standard relationships

$$Q_t = E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}\right] \tag{80}$$

$$\frac{W_t}{P_t} = -\frac{\partial U_t/\partial H_t}{\partial U_t/\partial C_t} \tag{81}$$

Where $M_{t+1} = \frac{\partial U_t/\partial C_{t+1}}{\partial U_t/\partial C_t}$ is the real stochastic discount factor. Equation (21) follows immediately from (81).

Let C^* denote the trend value of consumption, which follows a deterministic log-linear process with growth rate g. Let $\widetilde{C}_t = C_t/C_t^*$ denote the percentage deviation from this trend. Equation (22) then follows immediately from (77) and (80).

Adopting similar notation, define $\widetilde{V}_t^C = V_t^C/V_t^{C*}$. Expanding V_t^C using this decomposition,

$$V_t^C = \widetilde{V}_t^C V_t^{C*} = \left[(1-\beta)(\widetilde{C}_t C_t^*)^\rho + \beta\mathscr{R}_t\left(\widetilde{V}_{t+1}^C V_{t+1}^{C*}\right)^\rho\right]^{1/\rho} \tag{82}$$

Note that in the steady state this simplifies to

$$(V_t^{C*})^\rho = (1-\beta)(C_t^*)^\rho + \beta(V_{t+1}^{C*})^\rho \tag{83}$$

Iterating forward one period,

$$(V_t^{C*})^\rho = (1-\beta)(C_t^*)^\rho + \beta\left[(1-\beta)(C_t^*\exp[g])^\rho + \beta(V_{t+2}^{C*})^\rho\right]$$

Continuing in this way to the limit,

$$(V_t^{C*})^\rho = (1-\beta)(C_t^*)^\rho \sum_{h=0}^{\infty} \beta^h \exp[hg\rho]$$

or, calculating the infinite sum,

$$V_t^{C*} = \left(\frac{1-\beta}{1-\beta\exp[g\rho]}\right)^{1/\rho} C_t^*$$

This implies that $\log V_t^{C*}$ has the same growth rate as c_t^*, namely g. So we can rewrite (83) as

$$(V_t^{C*})^\rho = (1-\beta)(C_t^*)^\rho + \beta\exp[g\rho](V_t^{C*})^\rho$$

or

$$V_t^{C*} = \left[\frac{(1-\beta)(C_t^*)^\rho}{1-\beta\exp[g\rho]}\right]^{\frac{1}{\rho}}$$

Substituting this back into (82) and rearranging produces equation (23).

C.2 Firms

Output is given by the aggregation of differentiated goods indexed on the unit interval:

$$Y_t = \left(\int_0^1 Y(i)_t^{1-1/\epsilon} di\right)^{\epsilon/(\epsilon-1)} \tag{84}$$

with corresponding price index

$$P_t = \left(\int_0^1 P(i)_t^{1-\epsilon} di\right)^{1/(1-\epsilon)} \tag{85}$$

Demand functions for each good are given by

$$C(i)_t = \left(\frac{P(i)_t}{P(t)}\right)^{-\epsilon} Y_t \tag{86}$$

As is standard, the firm's problem, log-linearized around the steady state, results

in the following recursive solution for inflation:

$$\pi_t = (1-\theta)(\log P_t^{opt} - \log P_{t-1}) \tag{87}$$

where $1-\theta$ is the probability of price adjustment and P_t^{opt} the firm's optimal reset price. Firms choose $P(i)_t^{opt}$ to maximize the expected discounted value of profits, taking demand as given. For a firm resetting its price in period t, this problem is

$$\max_{P_t^{opt}} \sum_{s=1}^{\infty} \theta^k \mathrm{E}_t \left[\frac{M_{t+s}}{P_{t+s}} \left(P_t^{opt} Y(i)_{t+s} - \frac{W_{t+s}}{(1-\psi) A_{t+s} N(i)_{t+s}^{-\psi}} \right) \right] \tag{88}$$

subject to

$$Y(i)_{t+s} = \left(\frac{P_t^{opt}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \tag{89}$$

In the non-stochastic steady-state, $M_{t+s} = (\beta g)^s$.

The firm's problem results in the following log-linear condition for inflation:

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda \hat{\mu}_t, \tag{90}$$

where

$$\hat{\mu}_t = -\log\left(\frac{W_t}{P_t}\right) + a_t - \psi n_t + \log(1-\psi) - \log(M)$$

Adopting the first-order approximation of $\log(1-N_t)$ and plugging in labor demand from (20) gives

$$\begin{aligned} \hat{\mu}_t &= \log\left(\frac{\eta(1-\psi)}{(1-\eta)}\right) + \log(1-H_t^*) + \frac{H^*}{1-H^*}\log H^* \\ &\quad - \left[1 + \frac{H^*}{(1-H^*)(1-\psi)} + \frac{\psi}{1-\psi}\right](y_t - a_t) - \log(M_{t+1}) \end{aligned}$$

Under this approximation, the k-period SDF $M_{t,t+k}$ (in our notation) is equal to β^k in the trendless model. With trend growth, this value becomes $M_{t,t+k} = \beta^k \exp kg(\rho - 1)$. This has an effect on the calculation of λ, but we are not computing this value from structural parameters, so that part doesn't matter for us. It also changes our inflation equation to

$$\pi_t = \beta \exp[g(\rho-1)] E_t[\pi_{t+1}] - \lambda \hat{\mu}_t,$$

We can write steady-state inflation as

$$\pi^* = -\frac{\lambda\hat{\mu}^*}{1-\beta\exp[g(\rho-1)]}$$

This assumes that markups have a constant steady-state value (no trend). It is straightforward to verify this claim by noting that $\hat{\mu}_t$ in the steady-state is given by:

$$\hat{\mu}_t^* = \left(\frac{(\chi\rho)}{(1-\psi)}\right)a_t^* - \left(\frac{(\chi\rho)}{(1-\psi)} - \rho\right)c_t^* - \log(\frac{\theta_t\chi M}{1-\psi})$$

Given the trend growth rates of a, c, and $\log\theta$ we derived earlier, the trend growth rate of $\hat{\mu}_t$ is

$$\hat{\mu}_{t+1}^* - \hat{\mu}_t^* = \left(\frac{(\chi\rho)}{(1-\psi)}\right)g - \left(\frac{(\chi\rho)}{(1-\psi)} - \rho\right)g - \rho g = 0$$

So $\hat{\mu}^*$ is indeed constant (and therefore so is π^*).

Finally, we can rewrite the inflation equation as

$$\pi_t = \beta\exp[g(\rho-1)]E_t[\pi_{t+1}] - \lambda\left[\left(\frac{(\chi\rho)}{(1-\psi)}\right)\widetilde{a}_t - \left(\frac{(\chi\rho)}{(1-\psi)} - \rho\right)\widetilde{y}_t\right] - \lambda\hat{\mu}^*$$

or

$$\pi_t = \pi^* + \beta\exp[g(\rho-1)]E_t[\pi_{t+1} - \pi^*] - \nu\widetilde{a}_t + \kappa\widetilde{y}_t$$

where ν and κ are reduced-form parameters.

D Solution Method

A "solution" in our model consists of a set of rules relating the state of the economy to each of our recursively-determined endogenous variables (Consumption, Inflation and Lifetime Utility) which satisfy the equilibrium conditions. Specifically, let N_s be the number of continuous state variables summarizing the state of the economy and $S_t \in \mathbf{R^{N_s}}$ denote the state of the economy. Then a solution consists of mappings $\{f_1, f_2, f_3\}$ where

$$\begin{aligned} f_1 : \mathbf{R^{N_s}} \to \mathbf{R} \ s.t. \ \tilde{V^C}_t = f_1(S_t), \\ f_2 : \mathbf{R^{N_s}} \to \mathbf{R} \ s.t. \ \tilde{C}_t = f_2(S_t), \\ f_3 : \mathbf{R^{N_s}} \to \mathbf{R} \ s.t. \ \pi_t = f_3(S_t), \end{aligned} \tag{91}$$

and which satisfy the equilibrium conditions of equations 1,2 and 3.

To obtain our solution set we employ iterative methods. To employ these methods in practice we need to discretize the statespace S_t, over which we can approximate our set of functional solutions, and build an associated transition matrix Q. We begin with discretization of the statespace S_t. We partition our set of state variables into a set of N_1 variables which follow an ar(1) $\{\mathbf{z}_1, \mathbf{z}_2, ...\mathbf{z}_{N_1}\}$ and a set of N_2 variables $\{\mathbf{h}_1, \mathbf{h}_2, ...\mathbf{h}_{N_2}\}$ which follow an ar(1) but have stochastic variance governed by $\{\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_{N_2}\}$ respectively, where each follows an ar(1) itself.[29] For $\mathbf{z}_i \in \{\mathbf{z}_1, \mathbf{z}_2, ...\mathbf{z}_{N_1}\}$ we use the well-known Rouwenhorst to build a discrete grid $\hat{\mathbf{z}}_\mathbf{i}$ of length N_{z_i}. For $\mathbf{v}_i \in \{\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_{N_2}\}$, we start by using Rouwenhorst to build a discrete grid $\hat{\mathbf{v}}_\mathbf{i}$ of length N_{v_i}. However, since $\mathbf{v}_i$ governs the conditonal variance of $\mathbf{h}_i$, we must ensure that $\hat{\mathbf{v}}_\mathbf{i}$ remains strictly positive. Therefore, if $\hat{\mathbf{v}}_\mathbf{i}$ has only positive values then we use that grid. Otherwise, we keep the largest point of the chosen grid, and create a uniform grid of same length N_{v_i} but beginning at a number just above 0 rather than the negative number chosen by Rouwenhorst. For $\mathbf{h}_i \in \{\mathbf{h}_1, \mathbf{h}_2, ...\mathbf{h}_{N_2}\}$ we use a uniform grid of length N_{h_i} which runs from 3 conditional standard deviations below the unconditional mean to 3 conditional standard deviations above the unconditional mean - where the conditional standard deviation is conditional on the largest grid point in the associated state $\mathbf{v}_i$'s grid $\hat{\mathbf{v}}_\mathbf{i}$ governing $\mathbf{h}_i$'s variance. This is critical, because to accurately represent dynamics in the state variable governing $\mathbf{h}_i$'s stochastic variance, $\mathbf{v}_i$, there must be grid points $\hat{\mathbf{h}}_\mathbf{i}$ such that, for cases where $\mathbf{v}_i$ - and therefore the conditional standard deviation of $\mathbf{h}_i$ - is large, there is nonzero conditional likelihood of transitioning to those states. If grid points for $\hat{\mathbf{h}}_\mathbf{i}$ are truncated too low, then conditional on high values of $\hat{\mathbf{v}}_\mathbf{i}$, all of these points would lie to the left of where the mass of the distribution ought to be. As such, changes in $\hat{\mathbf{v}}_\mathbf{i}$ will not lead to equivalent changes in the numerical standard deviation of $\hat{\mathbf{h}}_\mathbf{i}$.

Given our set of grids ($\mathbf{Z} = \{\hat{\mathbf{z}}_\mathbf{1}, \hat{\mathbf{z}}_\mathbf{2}, ...\hat{\mathbf{z}}_{\mathbf{N_1}}\}$),($\mathbf{H} = \{\hat{\mathbf{h}}_\mathbf{1}, \hat{\mathbf{h}}_\mathbf{2}, ...\hat{\mathbf{h}}_{\mathbf{N_1}}\}$), and ($\mathbf{V} =$

[29] Note $N_1 + 2N_2 = N_s$

$\{\hat{\mathbf{v}}_{\mathbf{1}}, \hat{\mathbf{v}}_{\mathbf{2}}, ...\hat{\mathbf{v}}_{\mathbf{N_1}}\}$), we build our discrete grid $\hat{\mathbf{S}}$ as the Cartesian product of all our individual grids: That is

$$\hat{\mathbf{S}} = \{(z_1, z_2, ...z_{N_1}, h_1, h_2, ...h_{N_2}, v_1, v_2, ...v_{N_2}) | z_i \in \hat{\mathbf{z}}_i \in \mathbf{Z}, h_i \in \hat{\mathbf{h}}_{\mathbf{i}} \in \mathbf{H}, v_i \in \hat{\mathbf{v}}_{\mathbf{i}} \in \mathbf{V}\},$$

with length:

$$N = (\prod_{j}^{N_1} N_{z_j})(\prod_{q}^{N_2} N_{h_q})(\prod_{i}^{N_2} N_{v_i})$$

We turn now to building transition matrix Q. Generic transition probabilities are given by:

$$\begin{aligned} q_{ij} &= Prob\left[\hat{\mathbf{S}}_{\mathbf{t+1}} = s_j | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right] \\ &= \prod_{g=1}^{N_1} Prob\left[\hat{\mathbf{z}}_{gt+1} = s_j(\hat{\mathbf{z}}_g) | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right] \cdot \\ &\prod_{g=1}^{N_2} Prob\left[\hat{\mathbf{v}}_{gt+1} = s_j(\hat{\mathbf{v}}_g) | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right] \cdot \\ &\prod_{g=1}^{N_2} Prob\left[\hat{\mathbf{h}}_{gt+1} = s_j(\hat{\mathbf{h}}_g) | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right], \end{aligned}$$

where i, j index elements of the state space ($s_j, s_i \in \hat{\mathbf{S}}$) and $s_j(\hat{\mathbf{z}}_g)$ refers to the element of s_j corresponding to state $\hat{\mathbf{z}}_g$. Note that since the transition probabilities of $\mathbf{Z}$ and $\mathbf{V}$ are only conditionally dependent on lagged values, we have,

$$Prob\left[\hat{\mathbf{z}}_{gt+1} = s_j(\hat{\mathbf{z}}_g) | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right] = Prob\left[\hat{\mathbf{z}}_{gt+1} = s_j(\hat{\mathbf{z}}_g) | \hat{\mathbf{z}}_{gt} = s_i(\hat{\mathbf{z}}_g)\right] \ \forall \ g \in \{1, 2, ..., N_1\},$$

$$Prob\left[\hat{\mathbf{v}}_{gt+1} = s_j(\hat{\mathbf{v}}_g) | \hat{\mathbf{S}}_{\mathbf{t}} = s_i\right] = Prob\left[\hat{\mathbf{v}}_{gt+1} = s_j(\hat{\mathbf{v}}_g) | \hat{\mathbf{z}}_{gt} = s_i(\hat{\mathbf{v}}_g)\right] \ \forall \ g \in \{1, 2, ..., N_1\},$$

Therefore we can rewrite q_{ij} as follows:

$$\begin{aligned} q_{ij} = \prod_{g=1}^{N_1} Prob\left[\hat{\mathbf{z}}_{gt+1} = s_j(\hat{\mathbf{z}}_g) | \hat{\mathbf{z}}_{gt} = s_i(\hat{\mathbf{z}}_g)\right] \cdot \prod_{g=1}^{N_2} Prob\left[\hat{\mathbf{v}}_{gt+1} = s_j(\hat{\mathbf{v}}_g) | \hat{\mathbf{v}}_{gt} = s_i(\hat{\mathbf{v}}_g)\right] \cdot \\ \prod_{g=1}^{N_2} Prob\left[\hat{\mathbf{h}}_{gt+1} = s_j(\hat{\mathbf{h}}_g) | \hat{\mathbf{h}}_{gt} = s_i(\hat{\mathbf{h}}_g), \hat{\mathbf{v}}_{gt} = s_i(\hat{\mathbf{v}}_g)\right], \end{aligned}$$

The transition probability $q_{ij}$is formed by taking the product of the individual conditional transition probabilities of the state variables. For the AR(1) variables the conditional likelihood of transition to state $s_j(\hat{\mathbf{z}}_g)$ only depends on its lagged value and none of the other states. However, for the set $\mathbf{H}$, the probability of transitioning from one state to the next $\left[\hat{\mathbf{h}}_{gt+1} = s_j(\hat{\mathbf{h}}_g)|\hat{\mathbf{h}}_{gt} = s_i(\hat{\mathbf{h}}_g)\right]$ depends critically on the conditional variance at the time governed by $\hat{\mathbf{v}}_{gt} = s_i(\hat{\mathbf{v}}_g)$.

The Rouwenhorst method provides us with probabilities:

$$Prob\left[\hat{\mathbf{z}}_{gt+1} = s_j(\hat{\mathbf{z}}_g)|\hat{\mathbf{z}}_{gt} = s_i(\hat{\mathbf{z}}_g)\right],\ \forall\ g \in \{1, 2, ...N_1\}\ \forall\ (i, j) \in \{1, 2, ...N\}X\{1, 2, ...N\}$$

In principal, these come from the Rouwenhorst transition matrix for each $\hat{\mathbf{z}}_g \in \mathbf{Z}$. The same is true for all $\hat{\mathbf{v}}_g \in \mathbf{v}$ where $v_g > 0\ \forall\ v_g \in \hat{\mathbf{v}}_g$. In the case where we have to truncate the grid to be greater than zero, we compute the point on the conditional PDF of the transition state and re-normalize such that conditional probabilities sum to one. Specifically, consider the probobilty of transitioning from some $v_{gi} \in \hat{\mathbf{v}}_g$ to some $v_{gj} \in \hat{\mathbf{v}}_g$. We have:

$$Prob\left[\hat{\mathbf{v}}_{gt+1} = v_{gj}|\hat{\mathbf{v}}_{gt} = v_{gi}\right] = \frac{\Phi\left(\frac{v_{gj} - (1-\rho_{v_g})\delta_{v_g} + \rho_{v_g} v_{gi}}{\sigma_{v_g}}\right)}{\sum_{q=1}^{N_{v_g}} \Phi\left(\frac{v_{gq} - (1-\rho_{v_g})\delta_{v_g} + \rho_{v_g} v_{gi}}{\sigma_{v_g}}\right)},$$

where $\delta_{v_g}, \rho_{v_g}, \sigma_{v_g}$ govern the mean, persistence and conditional variation of $\mathbf{v}_g$. For,

$$Prob\left[\hat{\mathbf{h}}_{gt+1} = s_j(\hat{\mathbf{h}}_g)|\hat{\mathbf{h}}_{gt} = s_i(\hat{\mathbf{h}}_g), \hat{\mathbf{v}}_{gt} = s_i(\hat{\mathbf{v}}_g)\right],$$

we follow a similar procedure using the point on the standard normal pdf, conditional on the mean governed by $s_i(\hat{\mathbf{h}}_g)$ and variance governed by $s_i(\hat{\mathbf{v}}_g)$. Specifically, consider the probobilty of transitioning from some $h_{gi} \in \hat{\mathbf{h}}_g$ to some $h_{gj} \in \hat{\mathbf{h}}_g$ conditional on being in $s_i(\hat{\mathbf{v}}_g) \in \hat{\mathbf{v}}_g$. We have:

$$Prob\left[\hat{\mathbf{h}}_{gt+1} = h_{gj}|\hat{\mathbf{h}}_{gt} = h_{gi}, \hat{\mathbf{v}}_{gt} = s_i(\hat{\mathbf{v}}_g)\right] = \frac{\Phi\left(\frac{h_{gj} - (1-\rho_{h_g})\delta_{h_g} + \rho_{h_g} h_{gi}}{s_i(\hat{\mathbf{v}}_g)}\right)}{\sum_{q=1}^{N_{h_g}} \Phi\left(\frac{h_{gq} - (1-\rho_{h_g})\delta_{h_g} + \rho_{h_g} h_{gi}}{s_i(\hat{\mathbf{v}}_g)}\right)}.$$

We are now equipped with our discretization of the statespace S_t and our associated transition matrix Q. We then approximate our solution set of functions over the discretized grid by defining $\{\hat{\mathbf{f}}_1, \hat{\mathbf{f}}_2, \hat{\mathbf{f}}_3\}$ over our set of nodes $\hat{\mathbf{S}}$. Our goal is to employ time iteration as in Judd (1996). However, because we have inetia in the policy rule

(equation ((16)) our "exogenous" state space $\hat{\mathbf{S}}$ does not fully summarize the state of the economy which depends criticality on i_{t-1}. As such, we consider the "box" of Cartesian Product of $\hat{\mathbf{S}}$ and a grid of nodes $\hat{\mathbf{I}}$ for i_{t-1}. This is equivalent to defining $\{\hat{\mathbf{f}}_1, \hat{\mathbf{f}}_2, \hat{\mathbf{f}}_3\}$ for each grid point $\hat{i} \in \hat{\mathbf{I}}$.

To employ time iteration we need to compute expectations. Without inertia, this is straightforward. The conditional expectation of the output gap for example would simply be:

$$E_t[\tilde{C}_{t+1}] = dot(Q, \hat{\mathbf{f}}_2)$$

That is, the conditional expectation of the output gap is the dot product of the transition matrix with the vector which approximates consumption over the states. However, with inertia, this is no longer the case. Denote $\hat{\mathbf{f}}_2^{\hat{i}}$ as the approximation for the output gap over the set of states conditional on $i_{t-1} = \hat{i}$. The dot product above then yields the expecation condtional on $i_t = \hat{i}$:

$$dot(Q, \hat{\mathbf{f}}_2^{\hat{i}}) = E_t[\tilde{C}_{t+1} | i_t = \hat{i}]$$

We compute the above expectation for all nodes $\hat{i} \in \hat{\mathbf{I}}$, and build an interpolating function $h(\hat{i}) = E_t[\tilde{C}_{t+1} | i_t = \hat{i}]$ which provides the conditional expectation in each node conditional on $i_t = \hat{i}$. To compute the expectation of any endogenous variable at any node we evaluate this interpolated function at the interest rate observed in that node, which is given at all states through the taylor rule.

This procedure allows us to compute expectations and therefore we can employ time iteration as in Judd (1998). We do this for all macro variables and then the same for the Price-Dividend ratio for equities. Computing the term structure of interest rates does not require iteration.